

**COST OF CAPITAL, HURDLE RATE, AND DEFAULT RISK:
A ROBUST MODEL OF INTERNAL CAPITAL MARKETS^{†‡}**

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Abstract. I formulate a multi-division firm model endogenously connecting a firm's cost of capital to the hurdle rate for project selection, through the default probability of the firm. Other models typically take the cost of capital as given and examine how the hurdle rate or other variables of interest are linked to the cost of capital. In the model here, there is a transaction cost of financing, namely, the legal cost of collection in case of default. Banks set the loan interest rate taking into account the default risk factor, which can be affected by the number of projects financed by a firm. Anticipating the impact on the cost of capital (i.e., loan interest rate), the firm sets a hurdle rate accordingly to determine the number of projects to invest and finance. As this affects the default probability of the firm, the cost of capital and the hurdle rate are jointly determined. The model is built upon recent results on multi-unit auctions in the mechanism design literature. It demonstrates how a firm may operate an internal capital market (ICM) to elicit private information from division managers and allocate capital to their projects efficiently. The model is "robust" in the sense that it allows a state-by-state analysis based on any realized profile of managers' private information. As such, the class of mechanisms analyzed in the paper is easier to implement than those suggested by Bayesian analyses, which typically require correctly specifying a common prior. Getting the prior right is often difficult in practice. I use the model to understand why operating an ICM may be better than giving division managers complete autonomy in investing and financing decisions. Such reasons include a lower cost of capital and a smaller expected loss in shareholder value in case of default, both due to co-insurance that reduces the default probability. Under particular circumstances, the default probability of an ICM firm can be shown to take a geometric form and consequently the (inverse) capital "supply curve" is downward sloping. Interestingly, this leads to "capital sponsoring" with a subsidized hurdle rate, rather than capital rationing. (*JEL* G31, G33, D23, L22)

Keywords: Cost of capital, hurdle rate, default risk, project selection, project financing, capital rationing.

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Cost of Capital, Hurdle Rate, and Default Risk: A Robust Model of Internal Capital Markets

1. Introduction

In this paper, I formulate a model of a firm with multiple divisions. The firm operates an internal capital market (ICM) to elicit private information from division managers and allocate capital to their projects. With the model, I derive results linking the cost of capital endogenously to the hurdle rate for project selection through the default probability of the firm.

There have been many models of firms with multiple divisions. For example, in accounting, models have been proposed to examine how a transfer pricing system can be designed to allocate costs to divisions optimally (e.g., Melumad, Mookherjee, and Reichelstein [1992]). These models typically focus on operating activities of a firm and say little about their relation with the cost of capital. There are also multi-division models of capital budgeting that tie the hurdle rate for project selection to the cost of capital (e.g., Baldenius, Dutta, and Reichelstein [2007]). But the cost of capital is typically exogenously given in the models.¹ So a firm's cost of capital is not affected by the firm's "growth opportunities", e.g., the number of good projects it has in a year.

In finance, models of multiple divisions have been used to study ICM. A partial list includes Bernardo et al [2006, 2004], Inderst and Laux [2005], Ozbas [2005], Motta [2003], Inderst and Muller [2003], Stein [2002], Scharfstein and Stein [2000], Stein [1997], Gertner, Scharfstein, and Stein [1994]. These models either assume only one "winning" division can obtain financing from the firm, or every division will receive some fraction of the available capital. In practice, it is likely that divisions compete for financial resources in a way that is somewhere in between: not as tough as only one can win, nor as widely spread as every surely gets a bite.²

In contrast to others, the multi-division model of this paper endogenously links a firm's cost of capital to the number of projects invested by the firm, providing a micro-foundation to the relation between

¹ For multi-firm models, rather than multi-division firm models, an exception is Lambert, Leuz, and Verrecchia's [2007] study that also derives the cost of capital endogenously. But they use a different modeling approach based on the Capital Asset Pricing Model.

² In summarizing some survey evidence, Mukherjee and Henderson [1987] note that the reported project acceptance rate ranged from over 75% to over 90%. Segelod's [1995] field study provides some institutional details on the resource allocation process of a divisionalized firm.

financing and investing activities. On top of this basic objective, it is hoped that the model can be robust and tractable. Robustness gives some guarantee to the reliability of the results so that they are not sensitive to details of the model specification, such as a correctly specified prior belief about the (division) managers' private information. Tractability allows the model to serve as a simple baseline construct for further refinements or extensions so that a single family of models sharing the same structure can be used to understand a variety of corporate finance issues.

I view the model of this paper a normative one, in the sense that it illustrates how a firm can elicit private information from managers so as to allocate capital efficiently. Notwithstanding this, it can also have testable implications. For example, for firms following practices suggested by the model, one should expect to see more efficient investment across divisions than in others that allocate capital quite differently, of course assuming other factors are constant.

Figure 1 gives an overview of the important elements of the model. While I draw upon recent results on multi-unit auctions to build the model, I make no innovation in the multi-unit auction used as a mechanism to elicit private information and allocate capital. The use of the mechanism in the model is twofold. First, it converts an incomplete-information setting that is difficult to analyze into a more tractable complete-information setting, upon which more complex models can be built. Second, the conversion cannot be done in an arbitrary manner. To fulfill incentive compatibility and individual rationality constraints to be detailed in Section 3, the multi-unit auction has to take the form of an *ex post mechanism* of which a component can be naturally interpreted as a hurdle rate, which is widely used in practice. This and other structures of an *ex post mechanism* provide the basis for deriving other results in the paper, which are my contributions.

Through operating an ICM (i.e., using a multi-unit auction to elicit private information and allocate capital), the firm will become informed of the valuation profile μ that represents managers' private information about the expected returns of their projects. I assume that at no cost the firm can communicate μ to a bank. While unrealistic, I believe this is the best assumption to make at this stage of developing the model. Components of the model have existed in the literature for some time but assembling them to form this model is new. Given the complexity of the model, a more realistic assumption about how μ can be communicated will only confuse the readers more, instead of helping them to understand how the model functions. This does not mean future extensions or refinements of the model should continue to adopt this

unrealistic assumption. Indeed, as discussed in Section 4, the roles of auditor and analyst come into play when this restrictive assumption is relaxed, which is one of the reasons why the model is relevant to accounting beside its relevance to corporate finance.

In the model, internally generated cash flow is supposed to be insufficient to fund project investment. A second important simplifying assumption is that borrowing from a bank is the only financing source of the firm. While restrictive, it is not entirely unrealistic; there are big firms and numerous smaller firms in the world that are unlisted and rely heavily on bank loans to meet financing needs. Discussions in Section 4 touch on how relaxing this restrictive assumption may lead to stronger relevance of the model to accounting.

For simplicity, I suppose that banks have perfect access to capital at the market interest rate r , and there is perfect competition among banks. Given that μ can be communicated to a bank at no cost, perfect bank competition forces the loan interest rate c_0 (i.e., the firm's cost of capital for financing projects) to be set by adding on top of r a premium for default risk. The friction that drives the results of the model is a transaction cost of debt financing, namely the legal cost L of collection in case of default. For the shareholders, they also dislike default because there is a loss in shareholder value due to forced liquidation of assets. For simplicity, regularity conditions on parameters of the model (to be detailed in Section 3) are assumed such that the lending bank can always get full recovery of the loan amount plus interest in case of default. Therefore, the main driver of the default risk is the firm's default probability q_0 .

In general, the default probability can depend on μ , as suggested by the characterization provided in Section 3. However, in a later part of my analysis, I identify conditions under which the default probability takes a simple *geometric* form. Restricting only to geometric default probability highlights a sufficient condition for capital rationing not to occur in the model and clarifies the important role of the (inverse) capital "supply curve." Other models typically assume the cost of capital is given, which is equivalent to assuming a perfectly elastic capital "supply curve." The model here provides a micro-foundation demonstrating that default risk consideration can lead to a downward slopping capital "supply curve." Thereby it uncovers the rarely discussed incompatibility between a downward slopping capital "supply curve" and capital rationing.

More concretely, geometric default probability implies that the capital "supply curve" takes the functional form: $C(k)/k = (1+r) + Lq^k/k$, where q is the default probability of a division if it were to borrow

like a legally separate entity. This capital “supply curve” is associated with a “marginal cost” curve, $C'(k)$, everywhere below the “supply curve” such that $C'(k) < (1+r) < C(k)/k$ for any $k > 0$. My analysis shows that if confining to a simple class of ex post mechanisms, called *posted-price mechanisms*, the hurdle rate h will be set at a level such that $(1+h) = C'(A)$, where A is the number of projects to be financed. Note that not only is h a “subsidized” rate with respect to the cost of capital $c_0 = C(A)/A - 1$ but the rate is also “subsidized” even when compared to the market interest rate r . This “capital sponsoring” result is illustrated in Figure 2.

This paper makes six contributions. First, it provides a model connecting the cost of capital to the hurdle rate endogenously. Other models typically take the cost of capital as given and examine how the hurdle rate or other variables of interest are linked to the cost of capital. Second, the model fills a gap between two extreme types of models in the ICM literature, namely “only one can win” and “everyone gets a bite.” The allowing for financing genuinely multiple projects, ranging from one to all, is the reason why the cost of capital can be endogenously linked to the hurdle rate through the default probability. Third, the model provides a baseline construct with which extensions and refinements may be developed to examine other issues not analyzed in this paper (see discussions in Section 4). It offers the potential to organize the understanding of various related corporate finance and accounting issues using a single family of models all developed from the one here as a baseline construct.

Fourth, the analysis of the model adds to the understanding of why operating an ICM may be better than giving division managers complete autonomy in investing and financing decisions. Such reasons include a lower cost of capital and a smaller expected loss in shareholder value in case of default, both due to co-insurance that reduces the default probability. Fifth, the analysis also shows that with geometric default probability, the capital “supply curve” is downward sloping. This leads to the interesting phenomenon of a subsidized hurdle rate and “capital sponsoring,” which to my knowledge has not been examined in the literature before. Finally, the model demonstrates to practitioners how a firm may operate an ICM to elicit private information from division managers and allocate capital to their projects efficiently. Because the model is “robust” in the sense that it does not depend on detailed specifications of managers’ private information, the class of mechanisms suggested here is easier to implement than suggestions derived from a Bayesian framework. A Bayesian analysis usually requires correctly specifying a common prior belief, which is often difficult in practice.

The rest of the paper is organized as follows. In the next section, I give an overview of the model

features and main results. The model is formally introduced and analyzed in Section 3. There I use the model to understand why operating an ICM can be better than giving division managers complete autonomy in investing and financing decisions. Part of the analysis is about characterizing the default probability that affects a firm's cost of capital and is affected by the number of projects financed. Section 4 discusses the model's relevance to corporate finance and to accounting; some unanswered questions related to the model are also posted there. Section 5 reiterates key findings of the paper. Technical derivations and proofs are relegated to the Appendix.

2. Overview of Model Features and Main Results

2.1 Model Features

Many multi-division models have explored incentive issues related to getting managers to work hard (e.g., Stein [2002] and Bernardo et al [2004, 2006]). In contrast, I focus on the capital allocation problem and maintain tractability of the model by abstracting away from moral hazard due to hidden action. The fundamental incentive issue in the model is to get division managers to reveal truthfully their private information about expected project returns.

In practice, a firm might find it hard to have some reliable prior beliefs about expected project returns known only to managers. The model does not require a firm to form such beliefs before it can implement the capital allocation scheme suggested. It is a robust model in the sense that the results derived do not critically depend on details of such prior beliefs. Because of the robustness and accordingly the easiness in implementing the suggested scheme, practitioners might find the model more relevant than others recommending "optimal" schemes derived from a Bayesian framework.

Consistent with the trend of emphasizing robustness in modeling (e.g., Bergemann and Morris [2005]), I focus on direct-revelation mechanisms ensuring truth-telling is dominant-strategy incentive compatible (DIC) and that participation is ex post individually rational (EIR). Such mechanisms are called *ex post mechanisms*. To draw on Segal's [2003] characterization of ex post mechanisms, I assume each division needs exactly one unit of capital to finance its project. While stylized, this is consistent with often found non-scalability of investment projects.

Like many others in the literature, the model assumes risk neutrality. This is important because the model is built upon results on multi-unit auctions in the mechanism design literature. These results rely

heavily on a quasi-linear functional form of agents' preferences (i.e., managers' here). The recent characterization by Saitoh and Serizawa [2008] however opens up potential for relaxing this assumption to allow preferences exhibiting income effects. This could provide a venue for future research, widening the applicability of the model.

Residual income is a divisional performance evaluation measure widely taught in textbooks. In particular, specific versions of it, such as EVA[®], are highly advocated by consulting companies like Stern Stewart & Co. (Stewart, Ellis, and Budington [2002]). The model assumes managers' objectives are to maximize residual incomes of their projects, with the capital charges endogenously determined in the model, rather than computed from an exogenously given required rate of return. Similarly, the firm's CEO is assumed to maximize the value created from project investment, taking into account the cost of financing. This can be viewed as the firm's "residual income", a performance measure analogous to the divisional residual income of project investment.

Incentive compensations are determined by the bonus rates for the CEO and managers exogenously specified in the model. While this differs from the widely accepted optimal contracting approach, the constant bonuses rates will facilitate empirical work built around the model. By contrast, it is not easy for researchers to test implications of the optimal contracting literature. Real-life compensation contracts often have simple structures. Rarely will one find such contracts nicely align with fine-tuned optimal contracts suggested by the literature. As my focus is on the capital allocation problem, optimal compensation design issues are left aside to maintain tractability of the model.

In the model, borrowing from banks is supposed to be the only source of external financing. For tractability, perfect bank competition is assumed so that any surplus from improved default risk is fully captured by the firm through a lower cost of (debt) capital.

The model provides a framework for examining a number of interesting questions. For example, within a firm, the capital charge is only a nominal accounting charge that serves the purpose of performance evaluation and involves no actual payment. As such, there is no issue about whether a division is able to make the "payment" or not. Suppose instead divisions are organized as separate legal entities with direct access to external financing in the debt market. The "capital charge" becomes an actual payment for the loan amount plus interest, and complications like default risk will kick in. The intuition is that an ICM can add

value by pooling together divisions' business risks and thereby relaxing the payment constraint that gives rise to the default risk.

In analyzing the model, I contrast two types of firm with different degrees of decentralization:

- *ICM Firm*: Divisions have autonomy in operating decisions but investing and financing decisions are centralized.
- *“Autonomous” Firm*: Divisions have autonomy in investing and financing decisions, in addition to operating decisions.

An ICM firm has an “active” CEO operating an ICM. The project selection and financing policy used by the CEO is modeled as a direct-revelation mechanism. With the ICM, information asymmetry is resolved before approaching a bank for debt financing. I suppose that in discussing with a bank, the CEO can credibly communicate what she knows about the expected project returns at no cost.

For an “autonomous” firm without an “active” CEO, a bank must resolve the asymmetric information problem itself. To give direct external financing the best chance to dominate an ICM, I assume the bank can also use a direct-revelation mechanism to elicit private information from managers approaching it for project financing. The only exception is that “capital charges” to divisions now involve actual payments, rather than merely nominal accounting charges.

2.2 Main Results

Below are main results of the paper:

Ex post mechanism as a project selection and financing policy: Simply requiring truth-telling to be dominant-strategy incentive compatible and participation to be ex post individually rational imposes an interesting structure on a firm's or a bank's policy for project selection and financing. Specifically, the policy is characterized by a hurdle rate above which all projects with at least that level of expected returns will be selected and financed.

Posted-price mechanism as the policy used by banks under perfect bank competition: In the case of an “autonomous” firm, perfect bank competition forces the policy used by a bank to take the form of a *posted-price mechanism*. Under such a mechanism, the hurdle rate is set at the same level as the “total cost” (i.e., 1 plus loan interest rate c) charged to a division, which is given by

$$(1+c) \equiv (1+r) + qL,$$

where r is the market interest rate at which banks have perfect access to capital, q is the default probability of a division borrowing as a “standalone legal entity,” and L is the lending bank’s legal cost of collection in case of default.

Benefits of ICM: Because a firm needs to hire an “active” CEO to operating an ICM, whether it is better to organize as an ICM firm depends on how large this hiring cost is, which is characterized by the bonus rate for the CEO. Organizing as an ICM firm provides two potential benefits. By pooling together divisions’ business risks, it might reduce the default risk and thereby lower the cost of capital. Additionally, when the lending bank invokes legal procedures of collection, there is a loss in shareholder value due to forced liquidation of assets in place. So the shareholders also want to avoid default for their own sake. These benefits are independent of the collateral-based argument suggested by Stein [1997]. Taken together the benefits will outweigh the cost if the CEO’s bonus rate is sufficiently small.

Characterization of default probability: The default probability plays an important role in the model to link hurdle rate and cost of capital together endogenously. A characterization of the default probability adds understanding to what are driving the probability and other results of the paper. Such a characterization is provided.

Geometric default probability: Conditions are given under which the default probability takes the simple geometric form, i.e., q^k , where k is the loan amount borrowed by an ICM firm. Loosely speaking, it requires the bonus rates for the CEO and managers to be “small” and that there are only “a few” divisions in the firm. The simple structure of geometric default probability relies on the cash flow of only one “successful” project to make up for the shortfall of all other “unsuccessful” projects with zero realized returns. It would not be possible if there are too many divisions in the firm. Because the bonus to the CEO is based on the value created from project investment, when all projects are “unsuccessful,” a larger bonus rate essentially means a bigger “rebate” by the CEO to help shouldering the borrowing cost of the firm. If the bonus rate is too large, the “rebate” would be enough to ensure no default even when all projects are “unsuccessful.” For a similar reason, the bonus rate for managers must not be too large either.

Capital sponsoring and subsidized hurdle rate: In prevalent models in the literature, typically an upward sloping capital “supply curve” is assumed, or otherwise an exogenous cost of capital is assumed, which is equivalent to a perfectly elastic capital “supply curve.” Both assumptions are consistent with standard settings in economic theory. The model of this paper provides a micro-foundation for assuming

possibly a downward sloping capital “supply curve.” When the default probability is geometric, the “total cost” of borrowing $C(k) \equiv (1+r)k + Lq^k$ is strictly convex, with the “average cost” of borrowing $C(k)/k = (1+r) + Lq^k/k$ decreasing in the loan amount k (i.e., a downward sloping capital “supply curve). Moreover, the “marginal cost” of borrowing $C'(k) = (1+r) + Lq^k \ln(q)$ is everywhere below the “average cost” of borrowing. Consequently, the CEO would prefer “capital sponsoring” with a subsidized hurdle rate set at the level such that $(1+h) = C'(A)$, where A is the number of projects approved for financing. This hurdle rate h is a “subsidized” rate in the sense that $h = C'(A) - 1 < C(A)/A - 1$, which is the endogenously determined cost of capital in the model. This is in stark contrast to the typically assumed setting with a hurdle rate above a firm’s cost of capital and therefore not every positive NPV project is financed, a phenomenon referred to as capital rationing.

While unexpected, the “capital sponsoring” result driven by geometric default probability is not unreasonable. Given perfect bank competition, the geometric default probability is decreasing in the loan amount borrowed. With a subsidized hurdle rate, more projects would be financed than otherwise would have, which leads to a more favorable term in borrowing. The positive externality that results makes the subsidized hurdle rate sustainable.

3. Model and Analysis

3.1 Model Specifications

I consider the setting of a firm with n divisions ($n \geq 2$), each headed by a manager, competing for capital allocated from the corporate headquarters, headed by the CEO. Each division has a project requiring exactly 1 unit of capital to invest. The project of division i , if executed, will bring in some uncertain future cash flow R_i , with its mean μ_i known only by the manager. This expected project return is referred to as manager i ’s valuation.

It is common knowledge that managers’ valuations follow some joint distribution with the support of each marginal distribution equal to $(\underline{\mu}, \bar{\mu})$, where $0 \leq \underline{\mu} < \bar{\mu} < \infty$. This is all the CEO knows about the valuations. On the other hand, the CEO and managers commonly know that conditional on the valuation profile $\boldsymbol{\mu} = (\mu_1, \mu_2, \dots, \mu_n)$, the project returns R_i ’s are independently and identically distributed as follows:

$R_i = 0$ with probability $q \in (0, 1)$, and

$\Pr\{R_i < \underline{R} + z \mid \boldsymbol{\mu}\} = q + (1-q)G(z \mid \mu_i)$ for all $z \in [0, \bar{z})$,

where $0 < \bar{z} \leq \infty$, with $\underline{R} > 0$ and $G(0 | \mu_i) = 0$. The “probability density function” $g(z | \mu_i) = G'(z | \mu_i)$ is positive and continuous for $z > 0$, and as defined, $\int_z g(z | \mu_i) dz = \mu_i / (1 - q)$ so that $E[R_i | \mu_i] = \mu_i$.

Each division has some assets in place required for its regular operations, which will bring in a certain (net) cash flow regardless of what happens to the division’s project. For simplicity, assume each division has an identical cash flow from regular operations to be generated during the year, $M > 0$, and beginning-of-the-year book value of assets, $B > 0$.

Besides what might be later arranged for project financing, the firm has no liabilities. For simplicity, bank loan is assumed to be the only source available to the firm for financing projects. The loan amount plus interest, denoted by $(1 + c_0)K$, is referred to as the “total cost” of borrowing K units of capital, where $K \geq 1$. The interest rate c_0 at which the firm is able to borrow the loan amount K is referred to as its cost of (debt) capital.

Although the model may be viewed as representing a typical round of a multiple-period model, here I assume it is one-shot. The sequence of events happening in the model is as follows:

- At the beginning of a year, it is common knowledge that the project selection policy adopted by the firm will continue as usual, namely, given any profile μ of expected project returns reported by managers, the policy $\langle x(\cdot), t(\cdot) \rangle$ specifies an outcome constituted of a capital allocation $\mathbf{x} = (x_1, x_2, \dots, x_n) \in \{0, 1\}^n$, with $x_i = 1$ indicating the approval of division i ’s project, and a “payment” scheme $\mathbf{t} = (t_1, t_2, \dots, t_n) \in \mathbf{R}^n$, with t_i standing for an accounting charge to division i .³ Though not confined to be so at this point of the model specification, it will be clear shortly that t_i ’s are zero for divisions with “losing” projects, i.e., those not selected.
- It is also common knowledge that the firm’s compensation policy will continue as usual, namely:
 - “Losing” managers, whose projects are not selected, will receive a basic salary, which is normalized to zero for simplicity;
 - “Winning” manager i will receive a basic salary, also normalized to zero, plus a bonus, at the

³ Recall that 1 is the unit of capital invested in a project. So $(t_i - 1)/1$ is the required (rate of) return often discussed in accounting textbooks under the topic of capital budgeting.

rate of $\beta > 0$, based on the approved project's residual income $R_i - t_i$.⁴

- The CEO will receive a basic salary, also normalized to zero, plus a bonus, at the rate of $\beta_0 > 0$, based on the value $\sum_i R_i x_i - (1+c_0)K$ created from projects selected and financed at the cost of capital c_0 .⁵
- Given the project selection and compensation policies, each manager chooses whether to put (costless) effort to develop his project proposal. The effort exerted on planning for the project will raise the expected project return.⁶
- Managers privately learn about the expected returns of their projects and submit reports to convey the information to the CEO.
- Everyone learns about the market interest rate $r > 0$ at which banks can have perfect access to capital.
- At this point, a manager can “take the project with him” to pursue an outside option (e.g., seek venture capital to start his own company), which will give an expected payoff of no more than $\beta(\mu_i - c_0)$. So he stays. Similarly, the CEO can choose to start her own company and invite “winning” managers to join. This outside option will give her an expected payoff of no more than $\beta_0[\sum_i \mu_i x_i - (1+c_0)K]$. So she also stays.⁷
- The CEO contacts a banker to share the information about projects available. The banker makes an offer on the loan terms, namely the schedule of interest rate at each loan amount requested. The offer is competitive, and the CEO accepts it with the loan amount finalized.
- The CEO allocates the capital to the divisions and imposes accounting charges accordingly.

⁴ At the expense of more complicated notations, the bonus rate can be specified based on *post-bonus*, rather than pre-bonus, residual income. See related discussions on bonus compensation computation in accounting textbooks such as Kieso, Weygandt, and Warfield [2006] and Stice, Stice, and Skousen [2007].

⁵ At the expense of more complicated notations, the bonus rate for the CEO can be specified based on shareholder value, namely, the expected equity value at the year end after closing income to retained earnings. This alternative specification is consistent with a stock bonus. Some analyses of the paper however become intractable with this alternative specification.

⁶ This dummy event is included for future extensions to introduce moral hazard into the model.

⁷ This dummy event is included to justify non-zero bonus rates exogenously set in the model, which could be an endogenous outcome of expanded modeling.

- Each manager continues his division's regular operations from which a cash flow is generated. Alongside with the regular operations, "winning" managers execute the approved projects, with their returns realized some time before the year end. Compensations to the CEO and managers are paid at the year end accordingly.
- Following the year end, if the cash available is enough to cover the "total cost" of borrowing, the payment is made to the lending bank. Otherwise the firm defaults, and the bank seeks recovery by forced liquidation, which results in a legal cost of $L > 0$ to the bank. The liquidation value of the firm's assets is a fraction $\lambda \in (0, 1)$ of their beginning-of-the-year book value.

Managers are expected payoff maximizers. Conditional on the valuation profile $\boldsymbol{\mu}$, a manager's expected payoff is equivalent to

$$E[R_i x_i(\boldsymbol{\mu}) - t_i(\boldsymbol{\mu}) \mid \boldsymbol{\mu}] = \mu_i x_i(\boldsymbol{\mu}) - t_i(\boldsymbol{\mu}).$$

This quasi-linear functional form of managers' expected payoffs allows utilizing directly some results on multi-unit auctions in the mechanism design literature. In particular, the following definition and lemma are essentially due to Segal [2003]:

DEFINITION 1 (Segal [2003]): A mechanism $\langle \mathbf{x}(\cdot), \mathbf{t}(\cdot) \rangle$ is an *ex post mechanism* if it satisfies *dominant-strategy incentive compatibility* (DIC) and *ex post individual rationality* (EIR):

For any manager i , any valuation profile $\boldsymbol{\mu} \in (\underline{\mu}, \bar{\mu})^n$, and any $\hat{\mu}_i \in (\underline{\mu}, \bar{\mu})$,

$$[\text{DIC}]: \mu_i x_i(\boldsymbol{\mu}) - t_i(\boldsymbol{\mu}) \geq \mu_i x_i(\hat{\mu}_i, \boldsymbol{\mu}_{-i}) - t_i(\hat{\mu}_i, \boldsymbol{\mu}_{-i}),$$

$$[\text{EIR}]: \mu_i x_i(\boldsymbol{\mu}) - t_i(\boldsymbol{\mu}) \geq 0.$$

where $(\hat{\mu}_i, \boldsymbol{\mu}_{-i})$ means the valuation profile constructed from $\boldsymbol{\mu}$ by replacing μ_i with $\hat{\mu}_i$.

LEMMA 1 (Segal [2003]): A deterministic mechanism $\langle \mathbf{x}(\cdot), \mathbf{t}(\cdot) \rangle$ is an *ex post mechanism* if and only if for each manager there exist functions $p_i, s_i: (\underline{\mu}, \bar{\mu})^{n-1} \rightarrow \mathbf{R}_+$ such that for every valuation profile $\boldsymbol{\mu} \in (\underline{\mu}, \bar{\mu})^n$,

$$x_i(\boldsymbol{\mu}) = 1 \text{ if } \mu_i \geq p_i(\boldsymbol{\mu}_{-i}), x_i(\boldsymbol{\mu}) = 0 \text{ otherwise, and}$$

$$t_i(\boldsymbol{\mu}) = p_i(\boldsymbol{\mu}_{-i}) x_i(\boldsymbol{\mu}) - s_i(\boldsymbol{\mu}_{-i}).$$

In structuring a policy to allocate capital and charge for the use, the CEO is assumed to restrict attention to ex post mechanisms only, which do not require any knowledge about the distribution of $\boldsymbol{\mu}$. The CEO is also an expected payoff maximizer. Conditional on the valuation profile $\boldsymbol{\mu}$, her expected payoff is equivalent to

$$\sum_i \mu_i x_i(\boldsymbol{\mu}) - (1+c_0)K.$$

3.2 “Autonomous” versus ICM Firm

In an ICM firm that has an “active” CEO operating an ICM, information asymmetry is resolved before approaching a bank. Suppose it is prohibitively costly for the CEO to commit criminal acts like falsifying information in project proposals submitted by division managers. Then by demanding the CEO to provide the proposals to support her claim about $\boldsymbol{\mu}$, the information can be credibly communicated to a bank.

I suppose that in discussing with a bank, the CEO can credibly communicate $\boldsymbol{\mu}$ at no cost. With complete information and perfect competition from other banks, the bank will set c_0 to equate its cost of providing the loan. Suppose further that even in the case of default the firm has enough assets in place for the bank to get full recovery of the loan amount plus interest. Then c_0 will be set by adding to r a risk premium due to the expected loss as a result of the legal cost and the default probability:

$$c_0 = r + q_0(K, \boldsymbol{\mu})L/K,$$

where $q_0(K, \boldsymbol{\mu})$ denotes the default probability of the firm when K units of capital are borrowed to finance projects from a pool characterized by $\boldsymbol{\mu}$.

Instead of operating an ICM, the firm could have organized divisions in a fully autonomous fashion, namely, separately incorporating the divisions and delegating project selection and financing responsibilities to them. In such an “autonomous” firm, the CEO plays merely a “passive” role of ordinary administration and receives only a basic salary normalized to zero.

For simplicity, assume each division in the firm has enough assets in place for a lending bank to get full recovery of the loan amount plus interest. Then the expected cost of providing a loan to division i is given by

$$(1+c_i) \equiv (1+r) + L \Pr\{\text{default by division } i \mid \boldsymbol{\mu}\}.$$

Without an “active” CEO operating an ICM, a bank must resolve the asymmetric information problem itself. To give direct external financing the best chance to dominate an ICM, I assume the bank can

also use an ex post mechanism to elicit private information about $\boldsymbol{\mu}$. The only exception is that the $t_i(\boldsymbol{\mu})$'s charged to divisions now involve actual payments, rather than merely accounting charges.

A bank's objective is to choose an ex post mechanism $\langle \mathbf{x}(\cdot), \mathbf{t}(\cdot) \rangle$ to maximize its expected profit from lending,

$$\sum_i [t_i(\boldsymbol{\mu}) - (1+c_i)x_i(\boldsymbol{\mu})],$$

subject to the zero expected profit condition due to perfect bank competition:

$$\sum_i [t_i(\boldsymbol{\mu}) - (1+c_i)x_i(\boldsymbol{\mu})] = 0.$$

When an optimum is attained, no change in $\langle \mathbf{x}(\cdot), \mathbf{t}(\cdot) \rangle$ can lead to a positive expected profit to a bank.

Although a bank can guarantee a zero expected profit by refusing to issue any loan, this is not optimal if issuing some loans can create efficiency surplus. When such unexploited surplus exists, there would be a way for a competing bank to offer loan terms to realize the surplus and make a positive expected profit, which however cannot happen under perfect competition. That is to say, optimality together with competition requires a bank to issue loans efficiently, given the constraints imposed by an ex post mechanism and the legal cost of collection in case of default.

If ignoring the DIC and EIR constraints of an ex post mechanism, efficient loan issuance would imply

$$x_i(\boldsymbol{\mu}) = 1 \text{ if } \mu_i \geq (1+c_i), x_i(\boldsymbol{\mu}) = 0 \text{ otherwise.}$$

This, however, cannot constitute an ex post mechanism unless $(1+c_i) \equiv (1+r) + LPr\{\text{default by division } i \mid \boldsymbol{\mu}\}$ is unrelated to μ_i . The regularity condition below specifies circumstances under which $(1+c_i)$ is indeed unrelated to μ_i . When the condition is met, a simple *posted-price mechanism* is optimal to a bank. This result is given in Lemma 2.

REGULARITY CONDITION:

- (a) $G((1+r) + L - M/(1-\beta) \mid \mu_i) < 1$ for any μ_i ;
- (b) $(1+r) \leq (\lambda B + M)/(1-\beta) - L$;
- (c) $(1+r) \leq \underline{R} + M/(1-\beta) - L$;
- (d) $(M+L)/(1-\beta) \leq (1+r)$.

Part a of the regularity condition ensures that the default probability is not too close to 1. Part b of

the condition says the liquidation value λB of a division's assets is not too small to prevent full recovery of the loan amount plus interest when legal procedures are invoked. These two parts together with part c imply the default probability of a division is at most q . Finally, part d guarantees that the legal cost L is not too large to make invoking the legal procedures unwise.

LEMMA 2: Suppose the regularity condition holds. Then for a “winning” division (i.e., one provided with a loan), the default probability is q and hence its cost of capital is $c \equiv r + qL$; a *posted-price* mechanism $\langle \mathbf{x}(\cdot), \mathbf{t}(\cdot) \rangle$ with

$$\begin{aligned} x_i(\boldsymbol{\mu}) &= 1 \text{ and } t_i(\boldsymbol{\mu}) = (1+c) && \text{if } \mu_i \geq (1+c), \\ x_i(\boldsymbol{\mu}) &= t_i(\boldsymbol{\mu}) = 0 && \text{otherwise,} \end{aligned}$$

is optimal to a bank.

Suppose the regularity condition is met and thus the default probability of any division provided with a loan is q . If posted-price mechanisms are used by banks,

$$I(c) \equiv \{i \mid \mu_i \geq (1+c) \text{ for } \mu_i \text{ of } \boldsymbol{\mu}\}$$

would be the index set of the “winning” divisions provided with loans. Let K denote the number of “winning” projects, i.e., $K \equiv |I(c)|$. Conditional on $\boldsymbol{\mu}$, the expected equity value of an “autonomous” firm (without an ICM) is given by

$$[\text{AF}]: \quad nM - Kq(1-\lambda)B + (1-\beta)[\sum_{i \in I(c)} \mu_i - (1+c)K].$$

The first term of this expression is the cash flow generated from regular operations of all n divisions. The second term is the expected loss in shareholder value due to forced liquidation of assets in case of default. The last term is the part of value created from “winning” projects that is retained by shareholders. A fraction β of the value created is paid to “winning” managers as their bonuses.

For ease of exposition, suppose for the moment that a CEO operating an ICM also uses a posted-price mechanism with $t_i(\boldsymbol{\mu}) = p_i(\boldsymbol{\mu}_{-i}) = (1+c)$ and the loan amount she borrows is also K . Bank competition forces the interest rate offered to the CEO to be set at a level given by the zero expected profit condition, i.e.,

$$(1+c_0)K = (1+r)K + q_0(K, \boldsymbol{\mu})L,$$

where $q_0(K, \boldsymbol{\mu})$ is the default probability of the firm given that K units of capital are borrowed to finance projects from a pool characterized by $\boldsymbol{\mu}$. The way $q_0(K, \boldsymbol{\mu})$ is defined assumes that successful voluntary

liquidation of some assets to avoid default cannot be completed before the lending bank invokes the legal procedures to liquidate the firm.

Given the suppositions above, the following would be a “lower bound” of such an ICM firm’s expected equity value:

$$[\text{IF}]: \quad nM - q_0(K, \boldsymbol{\mu})(1-\lambda)nB + (1-\beta)[\sum_{i \in I(c)} \mu_i - (1+c)K] + (c - c_0)K - \beta_0[\sum_{i \in I(c)} \mu_i - (1+c_0)K].$$

This is a “lower bound” because the possibility of reorganization might result in a higher value to the shareholders. If liquidating some of the assets is sufficient to meet the payment obligation, the second term in the expression above would exaggerate the expected loss in shareholder value in case of default. The third and fourth terms in the expression add up to

$$[\sum_{i \in I(c)} \mu_i - (1+c_0)K] - \beta[\sum_{i \in I(c)} \mu_i - (1+c)K].$$

This is the value created by the “winning” projects minus the bonuses paid to managers given that “winning” divisions are charged at $(1+c)$, rather than the “average cost” of borrowing $(1+c_0)$. Finally, the last term in the lower bound has no counterpart in the expected equity value of an “autonomous” firm. The “passive” CEO of such a firm is presumed to receive only a basic salary normalized to zero. By contrast, the CEO of an ICM firm receives also a bonus based on the value she creates by operating an “active” ICM. This bonus is the last term in the lower bound.

3.3 Default Probability of ICM Firm

In general, an ICM firm might attain an expected equity value higher than the lower bound IF by charging “winning” divisions (possibly asymmetrically) more or less than $(1+c)$ and by borrowing a loan amount different from K , as long as such adjustments constitute an ex post mechanism. Raising the capital charges $t_i(\boldsymbol{\mu})$ ’s can save bonus expenses but the project selection cutoffs $p_i(\boldsymbol{\mu}_i)$ ’s might need to be tightened accordingly, which can reduce the total value created by “winning” divisions. Depending on the characteristics of the default probability $q_0(K, \boldsymbol{\mu})$, borrowing less might reduce the cost of capital c_0 so substantially that it compensates for the loss in value due to a smaller number of “winning” projects. Given these flexibilities in improving upon a posted-price mechanism with given c and K , there is room for an ICM firm to achieve a higher equity value than the lower bound IF.

In contrast, an “autonomous” firm’s equity value AF can be lower than an ICM firm’s lower bound IF. This may be best seen by examining their difference with the former deducted from the latter:

$$\Delta \equiv [Kq - nq_0(K, \boldsymbol{\mu})](1-\lambda)B + [Kq - q_0(K, \boldsymbol{\mu})]L - \beta_0[\sum_{i \in I(c)} \mu_i - (1+c_0)K].$$

Suppose the bonus rate β_0 for the CEO is sufficiently small and therefore the cost of hiring her to operate an ICM is arbitrarily negligible. Then as long as $q_0(K, \boldsymbol{\mu}) < qK/n$, the difference Δ can be made positive. In words, the decrease in expected loss in shareholder value due to default risk, i.e., $[Kq - nq_0(K, \boldsymbol{\mu})](1-\lambda)B$, together with the saving in cost of capital due to co-insurance among divisions, i.e., $[Kq - q_0(K, \boldsymbol{\mu})]L$, will be large enough to cover the cost of operating an ICM, i.e., $\beta_0[\sum_{i \in I(c)} \mu_i - (1+c_0)K]$. This result is stated below as Proposition 1.

PROPOSITION 1 (*Benefits of internal capital markets*): Suppose $q_0(K, \boldsymbol{\mu}) < qK/n$ and hence $c_0 < c$. For sufficiently small $\beta_0 > 0$, an ICM firm's expected equity value can exceed an "autonomous" firm's.

It should be emphasized that Proposition 1 is a "can-be" result. Since the CEO's and the shareholders' interests do not align with each other completely, the equity value of an ICM firm might fail to exceed an "autonomous" firm's if the task of operating an ICM is fully delegated to the CEO. However, it is conceivable that the shareholders might impose restrictions on the ex post mechanism used, e.g., require capital rationing while leaving the project selection decision to the CEO. Such measures might prevent the CEO's opportunistic behavior from jeopardizing the firm's equity value.

The behavior of the default probability $q_0(K, \boldsymbol{\mu})$ of an ICM firm in general can be quite complex. To get some sense of how it might behave, let's continue to suppose that the firm uses a posted-price mechanism with $t_i(\boldsymbol{\mu}) = p_i(\boldsymbol{\mu}_i) = (1+c)$ and the loan amount borrowed is $K \equiv |I(c)|$, where $I(c) \equiv \{i \mid \mu_i \geq (1+c)\}$ for μ_i of $\boldsymbol{\mu}$.⁸ Given these suppositions, the firm's year-end cash position would be

$$\begin{aligned} nM + \sum_{i \in I(c)} R_i - \beta[\sum_{i \in I(c)} R_i - (1+c)K] - \beta_0[\sum_{i \in I(c)} R_i - (1+c_0)K] \\ = nM + (1-\beta-\beta_0)[\sum_{i \in I(c)} R_i - (1+c)K] + (c-c_0)(1-\beta_0)K + (1+c_0)K. \end{aligned}$$

Default occurs when the cash position is insufficient to meet the firm's payment obligation, i.e., when

$$\sum_{i \in I(c)} R_i < K[(1+c) - (c-c_0)(1-\beta_0)/\alpha] - nM/\alpha,$$

⁸ This could be the case if the provision of capital follows the traditional price-based transfer pricing framework, where the transfer price of internally provided goods is set at the outside market price.

where $\alpha \equiv (1-\beta-\beta_0)$. Let S denote the number of “successful” projects, i.e. those with positive realized returns. Given S , let $J(S)$ denote an index set of the S “successful” projects. There are altogether $K!/S!(K-S)!$ such $J(S)$'s each indicating a specific combination of the projects that constitute the S “successful” projects. Let $\Omega(S)$ denote the set of all such $J(S)$'s. Finally, define $Z_i = (R_i - \underline{R})\mathbf{1}_{\{R_i > 0\}}$; in words, Z_i is the part of R_i exceeding \underline{R} . Conditional on $R_i > 0$, Z_i follows the distribution $G(z_i | \mu_i)$.

With these notations, the default probability can be expressed as follows:

$$\begin{aligned} \Pr\{\text{default} | \boldsymbol{\mu}\} &= \sum_s \Pr\{\text{default} | S = s, \boldsymbol{\mu}\} \Pr\{S = s | \boldsymbol{\mu}\} \\ &= \sum_s \sum_{J(s) \in \Omega(s)} \Pr\{\text{default} | J(s), S = s, \boldsymbol{\mu}\} (1-q)^s q^{K-s} \end{aligned}$$

where

$$\begin{aligned} \Pr\{\text{default} | J(s), S = s, \boldsymbol{\mu}\} &= \Pr\{Z_{J(s)} + s \underline{R} < K[(1+c) - (c - c_0)(1-\beta_0)/\alpha] - nM/\alpha | \boldsymbol{\mu}\} \end{aligned}$$

and $Z_{J(s)} = \sum_{i \in J(s)} Z_i$ is the sum of the independent random variables Z_i 's for those $i \in J(s)$.

The distribution function of $Z_{J(s)}$, which is referred to in probability theory as the convolution of the distribution functions $G(z_i | \mu_i)$'s, can depend on the μ_i 's in a non-trivial way. For tractability, I assume the probability distribution function of $Z_{J(s)}$ simply takes the form $G(z | \mu_{J(s)})$, where $\mu_{J(s)} = \sum_{i \in J(s)} \mu_i$.⁹ Given this assumption, the conditional probability $\Pr\{\text{default} | J(s), S = s, \boldsymbol{\mu}\}$ can be expressed as follows:

$$\begin{aligned} \Pr\{\text{default} | J(s), S = s, \boldsymbol{\mu}\} &= \Pr\{Z_{J(s)} < K[(1+c) - (c - c_0)(1-\beta_0)/\alpha] - nM/\alpha - s \underline{R} | \boldsymbol{\mu}\} \\ &= G(K[(1+c) - (c - c_0)(1-\beta_0)/\alpha] - nM/\alpha - s \underline{R} | \mu_{J(s)}) \\ &= G(f(q_0, s, K, c) | \mu_{J(s)}), \end{aligned}$$

where

$$f(q_0, s, K, c) = K[(1+c) - (c - c_0)(1-\beta_0)/\alpha] - nM/\alpha - s \underline{R}$$

and $c_0 = r + q_0(K, \boldsymbol{\mu})L/K$.

⁹ One example is that Z_i follows the gamma distribution $\text{Gam}(\mu_i, 1-q)$, which has mean $\mu_i/(1-q)$ and variance $\mu_i/(1-q)^2$. Then $Z_{J(s)}$ follows the distribution $\text{Gam}(\sum_{i \in J(s)} \mu_i, 1-q)$.

Below is the next main result of the paper, which provides an equation characterizing the default probability of an ICM firm.

PROPOSITION 2 (*Characterization of default probability*): Suppose that an ICM firm uses a posted-price mechanism with $t_i(\boldsymbol{\mu}) = p_i(\boldsymbol{\mu}_{-i}) = (1+c)$ and the loan amount borrowed is $K \equiv |I(c)|$, where $I(c) \equiv \{i \mid \mu_i \geq (1+c) \text{ for } \mu_i \text{ of } \boldsymbol{\mu}\}$. Moreover, for $s \geq 1$, suppose that the sum of the parts of positive realized returns over \underline{R} , namely $Z_{J(s)} \equiv \sum_{i \in J(s)} Z_i$, where $Z_i \equiv (R_i - \underline{R}) \mathbf{1}_{\{R_i > 0\}}$, follows the distribution $G(z \mid \mu_{J(s)})$, where $\mu_{J(s)} \equiv \sum_{i \in J(s)} \mu_i$; for $s = 0$, define instead $G(z \mid \mu_{\emptyset}) = 0$ for $z \leq 0$ and $G(z \mid \mu_{\emptyset}) = 1$ for $z > 0$. Then for any given $\boldsymbol{\mu}$ and c and the K so determined, the default probability $q_0(K, \boldsymbol{\mu})$ of the ICM firm is given by the equation below, provided it admits an interior solution in $[0, 1]$:

$$q_0 = \sum_s \sum_{J(s) \in \Omega(s)} G(f(q_0, s, K, c) \mid \mu_{J(s)}) (1-q)^s q^{K-s},$$

where

$$f(q_0, s, K, c) = K[(1+c) - (c - c_0)(1-\beta_0)/\alpha] - nM/\alpha - s\underline{R},$$

$$c_0 = r + q_0 L/K, \text{ and } \alpha = (1-\beta-\beta_0).$$

To understand more about the properties of $q_0(K, \boldsymbol{\mu})$, let's consider the special case of $K = n$. Given the complex “combinatorial” structure of the equation defining $q_0(K, \boldsymbol{\mu})$, little can be said even for this special case. However, if there are “not too many” divisions in the firm and the bonus rates β_0 and β for the CEO and managers are “not too large” (both made specific shortly), then $q_0(n, \boldsymbol{\mu})$ will take the simple form of q^n . This follows from the fact that $f(q_0, s, K, c)$ is decreasing in s . As a result, $f(q_0, 1, n, c) \leq 0$ implies $f(q_0, s, n, c) \leq 0$ for all $s \geq 1$ and hence $G(f(q_0, s, n, c) \mid \mu_{J(s)}) = 0$ for all $s \geq 1$. It can be shown that $f(q_0, 1, n, c) \leq 0$ and $f(q_0, 0, n, c) > 0$. As $G(z \mid \mu_{\emptyset}) = 1$ for $z > 0$, it follows that

$$\begin{aligned} q_0 &= \sum_s \sum_{J(s) \in \Omega(s)} G(f(q_0, s, n, c) \mid \mu_{J(s)}) (1-q)^s q^{n-s}, \\ &= G(f(q_0, 0, n, c) \mid \mu_{\emptyset}) q^n \\ &= q^n. \end{aligned}$$

This result is stated as the lemma below.

LEMMA 3: Suppose the regularity condition holds. In addition, $n \leq [\underline{R} - L/(1-\beta-\beta_0)]/[1+(1+r) - M/(1-\beta)]$ and $\beta_0 < (1-\beta)(1-q\beta)L/(M+L)$. Then given $\boldsymbol{\mu}$ and c , if $K = n$, $q_0(n, \boldsymbol{\mu}) = q^n$.

This simple structure of the default probability relies on the cash flow of only one positive realized return project to make up for the shortfall of all other zero realized return projects. It would not be possible if there are too many divisions in the firm. Recall that the bonus to the CEO is based on the value created from project investment. When all projects have zero realized returns, a larger bonus rate essentially means a bigger “rebate” by the CEO to help shouldering the borrowing cost of the firm. If the bonus rate is too large, the “rebate” would be enough to ensure no default even when all projects have zero realized returns. For a similar reason, the bonus rate for managers must not be too large either.

Suppose further that β_0 , β , and n are sufficiently small such that $(nM + q\beta L)/(1-\beta-\beta_0) \leq (1+r)$. Then the simple structure of default probability derived above carries over to any $K \in \{1, 2, \dots, n\}$. To see why, first note that given the additional assumption on n and β_0 ,

$$f(q_0, s, K, c) = \{K[(1+r)\alpha - q\beta L] - nM + q_0(1-\beta_0)L - s\underline{R}\}/\alpha$$

is increasing in K . It has been shown in the proof of Lemma 3 that $f(q_0, 1, n, c) \leq 0$. It follows that $f(q_0, 1, K, c) \leq 0$ for all $K \leq n$. The additional assumption also implies $f(q_0, 0, 1, c) > 0$. As $f(q_0, s, K, c)$ is increasing in K , $f(q_0, 0, K, c) > 0$ for all $K \leq n$. Together they imply $q_0(K, \boldsymbol{\mu}) = q^K$ for any $K \in \{1, 2, \dots, n\}$. This result is stated as the next proposition.

ASSUMPTION SBAFD1 (“*Small bonuses, a few divisions*” – first version): The bonus rates for the CEO and managers are “small”, and there are only “a few” divisions in the firm, i.e., β_0 , β , and n satisfy the following conditions:

$$n \leq [\underline{R} - L/(1-\beta-\beta_0)]/[(1+r) - M/(1-\beta)];$$

$$\beta_0 < (1-\beta)(1 - q\beta)L/(M+L);$$

$$(nM + q\beta L)/(1-\beta-\beta_0) \leq (1+r).$$

PROPOSITION 3 (*Geometric default probability*): Suppose the regularity condition holds and β_0 , β , and n satisfy the assumption SBAFD1. If an ICM firm uses a posted-price mechanism with $c = r + qL$ as the (rate of) “required return” and borrows $K \equiv |I(c)|$ as the loan amount, where $I(c) \equiv \{i \mid \mu_i \geq (1+c) \text{ for } \mu_i \text{ of } \boldsymbol{\mu}\}$, then the firm’s default probability is $q_0(K, \boldsymbol{\mu}) = q^K$. Consequently, its cost of capital is $c_0 = r + Lq^K/K$, and “total cost” of borrowing is $(1+c_0)K = (1+r)K + Lq^K$.

Given this simple structure of an ICM firm's default probability, the result in Proposition 1 can be strengthened. A specific threshold for the CEO's bonus rate can now be provided. The stronger result is stated as the proposition below.

PROPOSITION 4: Suppose the regularity condition and assumption SBAFD1 hold. Given μ and c , and the K so determined, if $2 \leq K \leq n \leq [2((1-\lambda)B + L)/q - L]/(1-\lambda)B$ and $\beta_0 \leq \beta_0^*$, where

$$\beta_0^* \equiv [(q - nq^K/K)(1-\lambda)B + (q - q^K/K)L]/(\bar{\mu} - (1+r) - Lq^K/K),$$

then the equity value of an ICM firm can exceed that of an "autonomous" firm.

This proposition relies on the geometric default probability derived earlier, which hinges on the possibility of using the cash flow of only one "successful" project to make up for the shortfall of all other "unsuccessful" projects. This however is impossible if $K = 1$. Because the geometric default probability also requires the firm to have at most "a few" divisions, its total cash flow from regular operations alone is insufficient to prevent default either. Given the presumption that all assets will be liquidated in case of default, there would be too much loss in shareholder value if n is large. Hence, the proposition also requires $n \leq [2((1-\lambda)B + L)/q - L]/(1-\lambda)B$.

3.4 Capital Sponsoring and Subsidized Hurdle Rate

So far, I have assumed an ICM firm uses a posted-price mechanism with $t_i(\mu) = p_i(\mu_i) = (1+c)$ and the loan amount borrowed is $K \equiv |l(c)|$, where $c = r + qL$ is the interest rate divisions of an "autonomous" firm can obtain from banks. The characterization of the default probability in Proposition 2 remains valid if c and K are replaced by h and $K(h)$ respectively, where h is a *hurdle rate* (of required return) used by the firm and $K(h) \equiv |l(h)|$ is the loan amount corresponding to the rate.¹⁰ That is, for any given μ and h and the $K(h)$ so determined, the default probability $q_0(K(h), \mu)$ of the ICM firm is given by the equation below, provided it admits an interior solution in $[0, 1]$:

$$q_0 = \sum_s \sum_{J(s) \in \Omega(s)} G(f(q_0, s, K(h), h) | \mu_{J(s)}) (1-q)^s q^{K(h)-s},$$

where

$$f(q_0, s, K(h), h) = K(h)[(1+h) - (h - c_0)(1-\beta_0)/\alpha] - nM/\alpha - s\underline{R},$$

¹⁰ Other results have specifically used $c = r + qL$ in the proofs and cannot be generalized trivially as this.

$$c_0 = r + q_0 L/K(h), \text{ and } \alpha = (1-\beta-\beta_0).$$

In addition, the result of a geometric default probability can be generalized to this more flexible posted-price mechanism, provided that the bonus rates for the CEO and managers are “small” and there are only “a few” divisions in the firm. This time, “small” and “a few” are in the following sense:

ASSUMPTION SBAFD2 (“*Small bonuses, a few divisions*” – second version): The bonus rates β_0 and β for the CEO and managers and the number n of divisions in the firm satisfy the following conditions:

$$n \leq [\underline{R} - L/(1-\beta-\beta_0)]/[(1+r) - M/(1-\beta)];$$

$$(nM + \bar{\mu} \beta)/(1-\beta-\beta_0) \leq (1+r).$$

PROPOSITION 5 (*Geometric default probability for general posted-price mechanism*): Suppose the regularity condition and assumption SBAFD2 hold. If an ICM firm uses a posted-price mechanism with h as the hurdle rate for project selection and $K(h) \equiv |I(h)|$ as the loan amount for project financing, where $I(h) \equiv \{i \mid \mu_i \geq (1+h) \text{ for } \mu_i \text{ of } \boldsymbol{\mu}\}$, then for any $h \geq 0$ with $K(h) \geq 1$, the firm’s default probability is $q^{K(h)}$. Consequently, its “total cost” of borrowing is $C(K(h))$, where $C(k) \equiv (1+r)k + Lq^k$.

Finally, it is interesting to examine whether the concern for default discussed here can naturally lead to capital rationing. With perfect bank competition, surplus from reduced default risk is completely captured by the firm. If it faces an upward sloping capital “supply curve,” the firm would be analogous to the buyer in the classic monopsony case. In such circumstances, it would be optimal for the buyer to restrain the quantity purchased (i.e., capital rationing in this context) to obtain a more favorable purchase price (i.e., a lower cost of capital).

There are four caveats to this conclusion. First, in the model here the capital needed for investment is a whole number. If an upward sloping capital “supply curve” is sufficiently flat, the gain from reducing the loan amount by one unit can be too little to justify the loss in expected project returns. Second, the capital “supply curve” need not be upward sloping and consequently capital rationing could reduce the value created from project investment. Third, from the shareholders’ perspective, they would like to balance the value created from project investment with the expected loss in shareholder value due to default. The latter depends on the default probability, which could decrease with the loan amount even when the capital “supply curve” is upward sloping. So shareholders might dislike capital rationing even when the CEO

prefers. Finally, the whole issue is further complicated by the saving in bonuses paid to managers when a higher hurdle rate is set for capital rationing. This alone benefits the shareholders but the CEO is indifferent.

When the default probability is geometric, the capital “supply curve” is downward sloping. Therefore, capital rationing is not preferred by the CEO. A seemingly unexpected finding is that she would set the hurdle rate below the cost of capital to “subsidize” divisions for project investment, resulting in some “capital sponsoring.” This however is reasonable given perfect bank competition and geometric default probability, which is decreasing in the loan amount borrowed. With a subsidized hurdle rate, more projects would be financed than otherwise would have, which leads to a more favorable term in borrowing. The positive externality that results makes the subsidized hurdle rate sustainable. This last result of the paper is stated below.

PROPOSITION 6 (*Capital sponsoring and subsidized hurdle rate*): Suppose an ICM firm uses a posted-price mechanism with h as the hurdle rate for project selection and $K(h) \equiv |I(h)|$ as the loan amount for project financing, where $I(h) \equiv \{i \mid \mu_i \geq (1+h) \text{ for } \mu_i \text{ of } \boldsymbol{\mu}\}$. If the default probability of the firm is geometric, i.e., $q_0(k, \boldsymbol{\mu}) = q^k$, then the “total cost” of borrowing $C(k) \equiv (1+r)k + Lq^k$ is strictly convex, with the “average cost” of borrowing $C(k)/k = (1+r) + Lq^k/k$ decreasing in the loan amount k . Moreover, the “marginal cost” of borrowing $C'(k) = (1+r) + Lq^k \ln(q)$ is everywhere below the “average cost” of borrowing. Consequently, the CEO prefers capital sponsoring with a subsidized hurdle rate set at $h = C'(A) - 1$, where $A \equiv \max\{a \mid \mu_{\omega(a)} \geq C'(a) \text{ for } a \leq n\}$ is the number of projects approved for financing, and $\omega(a)$ denotes the index of the a th highest μ_i of $\boldsymbol{\mu}$.

4. Discussions

4.1 Relevance to Corporate Finance

The model has potential to shed light on a variety of corporate finance issues, like capital rationing as a response to managerial overconfidence, “socialistic” capital allocation, value of corporate diversification, boundaries of the firm, and spinoff and acquisition decisions.

Capital rationing as a response to managerial overconfidence: With the model, I am able to highlight the relation between capital “supply curve” and project financing. The analysis on geometric default probability demonstrates that a downward sloping capital “supply curve” is incompatible with capital rationing. In reality, a downward sloping capital “supply curve” might not occur very often and therefore

capital rationing is much often seen than “capital sponsoring.” When capital rationing occurs in a firm with a downward sloping capital “supply curve,” does this mean the model developed here is wrong?

Not necessary. Stein [2003] points out that “[a] ... potentially very promising agency theory of investment builds on the premise that managers are likely to be overly optimistic about the prospects of those assets that are under their control.” (p. 123) It has been well recognized that overconfidence as a cognitive bias exists even in competitive business environments (e.g., Russo and Schoemaker [1992] and Zacharakis and Shepherd [2001]). What Stein has not pointed out is “inflated” hurdle rate could be a simple way to curb managerial overconfidence on expected project returns. This “overconfidence” explanation implies seemingly inefficient capital rationing could be an efficient way to correct for otherwise inefficient capital allocation. Taking this into account, capital rationing can arise even when a firm faces a downward sloping capital “supply curve.” The model thus suggests the following testable hypothesis: Managerial overconfidence on expected project returns is more likely to be found in firms imposing capital rationing even with a downward sloping capital “supply curve” than in those with an upward sloping curve.

“Socialistic” capital allocation: The model captures the very essence of potential lack of relation between current investment opportunities and past investment performance. Imagine the model is repeated twice to form a two-period model. Suppose (expected) project returns are independently distributed among divisions and across periods. Moreover, the return of a project in period 1 is spread out to both periods. Given the lack of relation between expected project returns in the two periods, a division with a high expected return project in period 1 is not particularly likely to have it again in period 2 and get the project funded. However, the high expected return project in period 1 is likely to result in a high realized return spilling over to period 2. Consequently, it might appear that a division with seemingly high investment efficiency in period 2 fails to prevent capital from flowing to another division with seemingly low investment efficiency – a phenomenon referred to as “socialistic” capital allocation.

Empirical studies on ICM typically compute investment efficiency of a division on an annual basis. Hypotheses on the direction of the capital flow are tested accordingly. This approach ignores the fact that (i) cash flows from past investments affect the investment efficiency computed for the current period and (ii) the capital flow should be related to current investment opportunities, not past investment performance. An overlapping generation model using the model of this paper as a building block can give guidance to

empirical work on ICM and shed light on the true reason behind the phenomenon of “socialistic” capital allocation.

Value of corporate diversification: Thought not analyzed in this paper, a firm might be able to better reduce its default risk and the cost of debt financing by diversifying into different business lines. In contrast, the business risks of divisions of a focused firm tend to be highly correlated (e.g., due to systematic industry-level risks), which limits the extent of diversifying the risks. An analysis along this line would need an extension of the model to allow for correlated project returns; distributions of the expected project returns however can still be independent.

Boundaries of the firm: When more and more business lines of different sectors are pooled together under a single roof, chances are there would be insufficient talents to manage each of them efficiently. This would pose a limit on the extent default risk can be reduced through bringing in more business lines of different sectors. The limit could be a factor determining the boundaries of the firm. An analysis along this line would need an extension of the model to allow for correlated project returns, as well as asymmetric expected project returns.

Spinoff and acquisition decisions: A diversified firm might be very careful in keeping optimal the combination of divisions in different sectors for the purpose of reducing default risk. But unanticipated external shocks to industry sectors can affect divisions’ expected project returns in such a way that some divisions are no longer worth being included in the conglomerate, or new targets should be acquired to form a better “portfolio” of divisions in different sectors. Because diversification is less costly when divisions in different sectors have similar expected project returns, it is conceivable that sometimes a “strong” division would be spun off, sometimes a “weak” one. What can be sure is that those retained under a single roof are more similar to each other in terms of expected project returns. Analogously, acquisitions are more likely when the acquirer’s divisions and the acquiree are more similar in their expected project returns. An extension of the model could also be used to examine such issues.

Diversification discount: Suppose a firm has specialty in its focused sector, which was why it focused on the sector in the first place. Then it is quite likely that it does not initially have comparative advantages in other sectors later brought in for the default risk reduction purpose; otherwise, it would have focused on those sectors. If divisions of the firm have high expected project returns to begin with, its cost of equity capital should be relatively low, and therefore it is not urgent to reduce the cost of debt capital either.

So diversification for default risk reduction purpose is not likely to occur at that moment. Later if there are adverse shocks to the firm's focused sector, expected project returns of its divisions go down and its cost of equity capital goes up. Then it is both more fruitful and more urgent to consider reducing default risk by diversification. If diversification takes place at this moment, it might seem that diversification leads to lower equity value when in fact it is lower equity value that makes diversification a more sensible option to consider. As laid out here, an extension of the model might be useful for understanding the diversification discount.

4.2 Relevance to Accounting

Although the model is formulated to understand ICM in particular and corporate finance issues in general, it is also relevant to several issues in accounting.

Roles of auditor and analyst: Can auditors and (financial) analysts play some roles in the model? It has been assumed that at no cost the CEO can communicate to a bank the expected project returns (i.e., valuation profile μ) she has learnt through operating an ICM. This is unlikely in reality. Even if the CEO indeed would be able to provide the managers' project proposals to support her claim on μ , a bank might find it more convenient to use some "summary statistics" in making the loan term decision. Analyst earnings forecasts may thus serve as a third-party source of evidence for assessing the truthfulness of the CEO's claim on μ .

By contrast, auditors' job might be literally interpreted as only verifying historical information and therefore is unrelated to the forward-looking information μ . However, suppose that the model is extended to an overlapping generation model where the realized return of a project is to arrive at two time points, one at the end of current year and the other the end of the next year. Then the auditor's verification of the firm's current-year earnings would be informative about the realized project returns of the next year. Consequently, it would affect the firm's default probability of the next year and hence its cost of capital (i.e., the loan interest rate it is able to get) at the beginning of the next year for the upcoming year's project financing. Analogously, the auditor's verification of last year's earnings would be relevant to determining the cost of capital for current year's project financing.

Even without extending the model this way, the one-shot model can provide a role for auditor if there is imperfect information about B and M , i.e., the assets in place and the cash flow from regular

operations, respectively. These parameters of the model in general can affect the default probability. When they are not known with certainty, a financial audit on the balance sheet and income statement of the firm can reduce the uncertainty about B and M and potentially affect the cost of capital through affecting the default risk assessment.

Experimental studies on ICM: In recent years, there is a growing interest in accounting and economics investigating whether behavioral factors like honesty, trust, fairness, etc can play a role in competitive business environments. For example, Bruggen and Luft [2008] experimentally examine how competition and honesty interact in the capital budgeting process of an ICM setting. Some of the predictions are derived from informal theorizing because they could not find any existing model that considers a varying degree of competition in ICM (i.e., whether only one, two, or all three of the divisions in their setting can “win” the capital for project investment). The model of this paper fills a gap between two extreme types of models in the literature, namely “only one can win” and “everyone gets a bit”. It provides a theoretical benchmark for comparing to behaviors of non-(purely-)economic agents in an ICM setting and could be useful to experimental studies looking at similar settings.

Performance evaluation measures: Although not analyzed in this paper, the model may be used to examine the relative merits of different accounting performance measures. For example, return on investment (ROI) is widely discussed in accounting textbooks. Can it be a useful alternative to the residual income (RI) presumed in the model? RI is often argued to be superior to ROI, yet the latter is believed to be useful for comparing divisions with very different sizes. If the model is extended to allow for divisions with asymmetric sizes, could ROI become superior to RI under particular circumstances?

Cost of capital and required rate of return: RI has been advocated as the correct approach to comparing projects for investment decisions. Yet the specification of the required rate of return remains problematic in practice. Some have argued it should be set at the WACC, which is a widely used estimate of a firm’s cost of capital. The model here provides a micro-foundation to the linkage between the required rate of return (i.e., the hurdle rate h) and the “marginal cost” of borrowing (i.e., $C'(k)$) to which the cost of capital (i.e., $C(k)/k - 1$) is closely tied. Further improvement on the model to incorporate stock price for shares of the firm would clear up the role of WACC in this setting.

Accounting ratios and cost of capital: The model offers potential to build a micro-foundation to the linkage between widely used accounting ratios and cost of capital. For example, the equity value of an ICM

is affected by the book value of assets in place and the default probability, with the latter tied to the cost of capital. This might shed light on the linkage between book-to-market ratio and cost of capital. If the model can be generalized to let the book value of assets and cash flow from regular operations affect the default probability, it might shed light on the linkage between leverage ratio (or current ratio, etc) and cost of capital as well. But these are unrelated when restricted to geometric default probability.

4.3 Unanswered Questions

The analysis in this paper is only the first step in exploring the potential of the model and applying it to understand some of the corporate finance issues that may be examined with the model. A number of open questions remain.

Default probability: The analysis of the model is substantially simplified by the assumption of a specific distributional structure of the project returns. The probability mass at the lower end of the distribution's support and its disconnection from the smooth part of the distribution is critical to establishing a default probability unrelated to the expected returns of projects in divisions of an "autonomous" firm. This in turn allows a simple characterization of the cost of capital charged to the divisions. Can alternative assumptions on the distributional structure also lead to tractable analyses?

CEO's preference: Can the quasi-linear objective function of the CEO be formally justified by a career-concern or other types of models? Ignoring the cost-of-borrowing component of the CEO's objective function will lead to an often assumed preference for empire building (based on gross output). How would this alternative assumption change the results here?

Division managers' preferences: That a manager cares about the residual income of his division's project is important to making the model work. Otherwise, the accounting charges would have no impact on managers' behaviors, and the model would fall apart. Can alternative preferences be assumed to make the model work for other performance evaluation measures?

Compensation contracts observed in practice: The model as it stands excludes incentive compensations that are found in practice, e.g., budget-based bonus schemes (Sprinkle, Williamson, and Upton [2008] and Murphy [2001]). Generalizing the model to allow for such possibilities would provide interesting venues for future research.

Other roles of the headquarters: The model assumes a highly decentralized firm environment, where the only role of the corporate headquarters (HQ), headed by the CEO, is to allocate capital raised from

outside. It ignores the monitoring role the HQ can play and related control right issues that have been extensively studied in the literature (e.g., Gertner, Scharfstein, and Stein [1994], Scharfstein and Stein [2000], and Stein [1997, 2002]). It is certainly interesting to generalize the model to include monitoring activities and control rights of the HQ.

Non-geometric default probability: The characterization provided in the paper admits a wide range of default probability besides geometric. Can other assumptions be made to restrict the structure of default probability to other tractable forms? Advances in this direction would widen applications of the model.

5. Summary of Key Findings

Posted-price mechanism as a project selection and financing policy: Under perfect bank competition, the project selection and financing policy used by a bank takes the form of a posted-price mechanism, namely the hurdle rate is set at the same level as the “total cost” (i.e., 1 plus loan interest rate) charged to a division, with the risk premium over the market interest rate set according to the default probability and the legal cost of collection in case of default.

Benefits of ICM: Organizing as an ICM firm reduces default probability, which has two potential benefits: (i) lower cost of capital and (ii) smaller expected loss in shareholder value due to default. These benefits will outweigh the cost of hiring the CEO to operate the ICM if the bonus rate for the CEO is sufficiently small.

Default probability endogenously links together cost of capital and hurdle rate of an ICM firm.

Geometric default probability: When there are only “a few” divisions in the firm and the bonus rates for the CEO and managers are “small,” the default probability takes a geometric form.

Capital sponsoring and subsidized hurdle rate: A downward sloping capital “supply curve” is incompatible with capital rationing. With geometric default probability, the capital “supply curve is downward sloping, leading to “capital sponsoring” with a subsidized hurdle rate set below the endogenously determined cost of capital.

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APPENDIX

PROOF OF LEMMA 2: I will first show that under the regularity condition, the default probability of a “winning” division (i.e., one provided with a loan) is q , regardless of $\boldsymbol{\mu}$. Consequently, $t_i(\boldsymbol{\mu})$ must be set at $(1+c) \equiv (1+r) + qL$ for a “winning” division. Requirements of an ex post mechanism then imply it is weakly better to set $p_i(\boldsymbol{\mu}_{-i}) = (1+c)$. As a result, the requirements also imply $t_i(\boldsymbol{\mu}) = 0$ for a “losing” division (i.e., one declined for a loan), which would complete the proof.

First, note that given any ex post mechanism used by a bank, managers of the divisions will report their μ_i 's truthfully. Moreover, bank competition ensures that $t_i(\boldsymbol{\mu}) \leq (1+r) + L$ for a “winning” division. Otherwise, a competing bank could have offered an ex post mechanism with $t_i(\boldsymbol{\mu}) = (1+r) + L$ for a “winning division.” Any division willing to accept $t_i(\boldsymbol{\mu}) > (1+r) + L$ will find this alternative offer more attractive. Since division i 's year-end cash position, with bonus to the manager already paid, is $M + R_i - \beta(R_i - t_i(\boldsymbol{\mu}))$, parts a and b of the regularity condition imply that (i) the probability of default is less than 1, and (ii) if legal procedures of collection are invoked, the competing bank will surely get back $t_i(\boldsymbol{\mu}) = (1+r) + L$ for any realized R_i leading to default. Consequently, charging $t_i(\boldsymbol{\mu}) = (1+r) + L$ will lead to a positive expected profit for the competing bank, which violates the condition for perfect competition. Hence, the charge $t_i(\boldsymbol{\mu})$ to a “winning” division must not exceed $(1+r) + L$.

Now recall that the year-end cash position of a division is $M + R_i - \beta(R_i - t_i(\boldsymbol{\mu}))$. This will be sufficient to meet the payment obligation to the bank if $M + R_i - \beta(R_i - t_i(\boldsymbol{\mu})) \geq t_i(\boldsymbol{\mu})$, which is equivalent to $t_i(\boldsymbol{\mu}) \leq R_i + M/(1-\beta)$. With probability $1 - q$, the right hand side of this inequality is at least $\underline{R} + M/(1-\beta)$. Under part c of the regularity condition, this is no less than $(1+r) + L$, which in turn is no less than $t_i(\boldsymbol{\mu})$. Thus, the default probability of a division must be at most q , regardless of $\boldsymbol{\mu}$. Bank competition thus implies $t_i(\boldsymbol{\mu}) \leq (1+r) + qL$ for a “winning” division.

With probability q , $R_i = 0$, and division i 's year-end cash position is only $M + \beta t_i(\boldsymbol{\mu})$. For the division to meet its payment obligation, it requires that $t_i(\boldsymbol{\mu}) \leq M/(1-\beta)$. Note that $t_i(\boldsymbol{\mu}) \geq (1+r)$, which is a floor of the cost of lending, even without default risk. Under part d of the regularity condition, $M/(1-\beta) < (1+r) \leq t_i(\boldsymbol{\mu})$. Thus, the division will default when $R_i = 0$. In other words,

$$\Pr\{\text{default by division } i \mid \boldsymbol{\mu}\} = q.$$

Now suppose $t_i(\boldsymbol{\mu}) < (1+r) + qL$ for a “winning” division. This will result in a negative expected profit to a bank unless invoking legal procedures in case of default is unwise sometimes. However, this cannot be under part d of the regularity condition. To see this, recall that default occurs only when $R_i = 0$, which means the cash position of the division is only $M + \beta t_i(\boldsymbol{\mu})$. For this to be as great as the bank's payoff from invoking the legal procedures, it requires $t_i(\boldsymbol{\mu}) \leq (M+L)/(1-\beta)$. Since $t_i(\boldsymbol{\mu}) \geq (1+r)$, the inequality cannot hold under part d of the regularity condition. Therefore, it is always wise to invoke legal procedures in case

of default. As a result, setting $t_i(\boldsymbol{\mu}) < (1+r) + qL$ for a “winning” division will lead to a negative expected profit to a bank. This means only ex post mechanisms with $t_i(\boldsymbol{\mu}) = (1+c) \equiv (1+r) + qL$ for a “winning” division will be used.

Recall that for $\langle \mathbf{x}(\cdot), \mathbf{t}(\cdot) \rangle$ to be an ex post mechanism, there must exist $p_i, s_i: (\underline{\mu}, \bar{\mu})^{n-1} \rightarrow \mathbf{R}_+$ such that for every $\boldsymbol{\mu} \in (\underline{\mu}, \bar{\mu})^n$,

$$\begin{aligned} x_i(\boldsymbol{\mu}) &= 1 \text{ if } \mu_i \geq p_i(\boldsymbol{\mu}_{-i}), x_i(\boldsymbol{\mu}) = 0 \text{ otherwise, and} \\ t_i(\boldsymbol{\mu}) &= p_i(\boldsymbol{\mu}_{-i}) x_i(\boldsymbol{\mu}) - s_i(\boldsymbol{\mu}_{-i}). \end{aligned}$$

For a “winning” division, $t_i(\boldsymbol{\mu}) = (1+c)$. Since $s_i(\boldsymbol{\mu}_{-i}) \geq 0$, it follows that $p_i(\boldsymbol{\mu}_{-i}) \geq (1+c)$.

Suppose the loan approval cutoff $p_i(\boldsymbol{\mu}_{-i})$ is chosen to be above $(1+c)$. Then potential surplus from a division with μ_i such that $(1+c) < \mu_i \leq p_i(\boldsymbol{\mu}_{-i})$ would not be realized under such a mechanism. A competing bank could have used an ex post mechanism with $t_i(\boldsymbol{\mu})$ set strictly in between $(1+c)$ and $p_i(\boldsymbol{\mu}_{-i})$ and an approval cutoff identical to this $t_i(\boldsymbol{\mu})$. This competing bank would then do at least as good as the bank setting $p_i(\boldsymbol{\mu}_{-i}) > (1+c)$ and possibly better when indeed some divisions have μ_i 's falling in between this competing bank's $t_i(\boldsymbol{\mu})$ and the other bank's $p_i(\boldsymbol{\mu}_{-i})$. In short, setting $p_i(\boldsymbol{\mu}_{-i}) > (1+c)$ is weakly dominated. Consequently, an ex post mechanism with $p_i(\boldsymbol{\mu}_{-i}) = (1+c)$ is as good as any other ex post mechanisms.

Given that $p_i(\boldsymbol{\mu}_{-i}) = t_i(\boldsymbol{\mu}) = (1+c)$ for a “winning” division, it must be that $s_i(\boldsymbol{\mu}_{-i}) = 0$ for $\mu_i \geq (1+c)$. However, $s_i(\boldsymbol{\mu}_{-i})$ cannot depend on μ_i . It follows that $s_i(\boldsymbol{\mu}_{-i}) = 0$ even for $\mu_i \leq (1+c)$.

In conclusion, a posted-price mechanism with

$$\begin{aligned} x_i(\boldsymbol{\mu}) &= 1 \text{ and } t_i(\boldsymbol{\mu}) = (1+c) && \text{if } \mu_i \geq (1+c), \text{ and} \\ x_i(\boldsymbol{\mu}) &= t_i(\boldsymbol{\mu}) = 0 && \text{otherwise} \end{aligned}$$

is at least as good as any other ex post mechanisms to a bank. ■

PROOF OF LEMMA 3: As explained in the main text, it suffices to show that $f(q_0, 1, n, c) \leq 0$ but $f(q_0, 0, n, c) > 0$. First, note that $c = r + qL$ and when $K = n$, $c_0 = r + q_0L/n$. Thus,

$$\begin{aligned} f(q_0, 0, n, c) &= n[(1+c) - (c - c_0)(1-\beta_0)/\alpha] - nM/\alpha \\ &= [(1+c)\alpha - (c - c_0)(1-\beta_0) - M]n/\alpha \\ &= \{(1+r)\alpha + [q\alpha - (q - q_0/n)(1-\beta_0)]L - M\}n/\alpha \\ &= \{(1+r)\alpha + [(q_0/n)(1-\beta_0) - q\beta]L - M\}n/\alpha, \end{aligned}$$

which is positive if

$$(1+r) > [M + (q\beta - (q_0/n)(1-\beta_0))L]/\alpha.$$

To show that this inequality indeed holds, note that

$$\begin{aligned} \alpha(M+L) - (1-\beta)[M + (q\beta - (q_0/n)(1-\beta_0))L] \\ = (1-\beta)(M+L) - \beta_0(M+L) - (1-\beta)M - (1-\beta)(q\beta - (q_0/n)(1-\beta_0))L \end{aligned}$$

$$\begin{aligned}
&= (1-\beta)[1-q\beta + (q_0/n)(1-\beta_0)]L - \beta_0(M+L) \\
&\geq (1-\beta)(1-q\beta)L - \beta_0(M+L) \\
&> 0,
\end{aligned}$$

where the last inequality is due to the given assumption that $\beta_0 < (1-\beta)(1-q\beta)L/(M+L)$. Hence,

$$(M+L)/(1-\beta) > [M + (q\beta - (q_0/n)(1-\beta_0))L]/\alpha.$$

By part d of the regularity condition,

$$\begin{aligned}
(1+r) &\geq (M+L)/(1-\beta) \\
&> [M + (q\beta - (q_0/n)(1-\beta_0))L]/\alpha,
\end{aligned}$$

which implies $f(q_0, 0, n, c) > 0$.

Now consider

$$\begin{aligned}
f(q_0, 1, n, c) &= n[(1+c) - (c - c_0)(1-\beta_0)/\alpha] - nM/\alpha - \underline{R} \\
&= f(q_0, 0, n, c) - \underline{R},
\end{aligned}$$

which is non-positive if $f(q_0, 0, n, c) \leq \underline{R}$. Note that the assumption $n \leq (\underline{R} - L/\alpha)/[(1+r) - M/(1-\beta)]$ implies

$$n \leq \underline{R}/[(1+r) - M/(1-\beta) + L/n\alpha].$$

Thus,

$$\begin{aligned}
&f(q_0, 0, n, c) \\
&= \{(1+r)\alpha + [(q_0/n)(1-\beta_0) - q\beta]L - M\}n/\alpha \\
&\leq \{(1+r)\alpha + [(q_0/n)(1-\beta_0) - q\beta]L - M\} \underline{R}/[(1+r) - M/(1-\beta) + L/n\alpha]\alpha \\
&= \underline{R} \{(1+r)\alpha + [(q_0/n)(1-\beta_0) - q\beta]L - M\}/[(1+r) - M/(1-\beta) + L/n\alpha]\alpha.
\end{aligned}$$

To see that $\{(1+r)\alpha + [(q_0/n)(1-\beta_0) - q\beta]L - M\}/[(1+r) - M/(1-\beta) + L/n\alpha]\alpha < 1$, consider

$$\begin{aligned}
&(1+r)\alpha + [(q_0/n)(1-\beta_0) - q\beta]L - M - [(1+r) - M/(1-\beta) + L/n\alpha]\alpha \\
&= -[1 - q_0(1-\beta_0)]L/n - q\beta L - \beta_0 M/(1-\beta) \\
&< 0.
\end{aligned}$$

Since $\{(1+r)\alpha + [(q_0/n)(1-\beta_0) - q\beta]L - M\} > 0$, it follows that $f(q_0, 0, n, c) < \underline{R}$.

As $f(q_0, 1, n, c) \leq 0$ and $f(q_0, 0, n, c) > 0$, $q_0(n, \mu) = q^n$. ■

PROOF OF PROPOSITION 3: As explained in the main text, it suffices to show that $f(q_0, 0, 1, c) > 0$. Note that

$$f(q_0, 0, 1, c) = [(1+r)\alpha - q\beta L - nM + q_0(1-\beta_0)L]/\alpha,$$

which is clearly positive given the additional assumption on n and β_0 . ■

PROOF OF PROPOSITION 4: Given the regularity condition and assumption SBAFD1, $q_0(K, \mu) = q^K$ and $(1+c_0)K = (1+r)K + q^K L$. Therefore, the difference obtained by deducting an ‘‘autonomous’’ firm’s

equity value AF from an ICM firm's equity value lower bound IF is as follows:

$$\Delta \equiv [Kq - nq^K](1-\lambda)B + [Kq - q^K]L - \beta_0[\sum_{i \in I(c)} \mu_i - (1+r)K - Lq^K].$$

First, recognize that $\bar{\mu}K > \sum_{i \in I(c)} \mu_i > (1+c)K = [(1+r) + qL]K \geq (1+r)K + q^K L$. Since

$$\beta_0^* \equiv [(q - nq^K/K)(1-\lambda)B + (q - q^K/K)L]/(\bar{\mu} - (1+r) - Lq^K/K),$$

it follows that $\beta_0^*(\bar{\mu}K - (1+r)K - q^K L) = (Kq - nq^K)(1-\lambda)B + (Kq - q^K)L$, which is positive under the condition that $2 \leq K \leq n \leq [2((1-\lambda)B + L)/q - L]/(1-\lambda)B$. For $\beta_0 \leq \beta_0^*$,

$$\begin{aligned} & \beta_0[\sum_{i \in I(c)} \mu_i - (1+r)K - q^K L] \\ & \leq \beta_0^*[\sum_{i \in I(c)} \mu_i - (1+r)K - q^K L] \\ & < \beta_0^*(\bar{\mu}K - (1+r)K - q^K L) \\ & = (Kq - nq^K)(1-\lambda)B + (Kq - q^K)L, \end{aligned}$$

which means $\Delta > 0$. ■

PROOF OF PROPOSITION 5: Note that

$$\begin{aligned} F(q_0, s, h) & \equiv f(q_0, s, K(h), h) \\ & = K(h) [(1+c_0)(1-\beta_0) - (1+h)\beta]/\alpha - nM/\alpha - s\underline{R} \\ & = K(h)[(1+r)(1-\beta_0) - (1+h)\beta]/\alpha + q_0 L(1-\beta_0)/\alpha - nM/\alpha - s\underline{R}. \end{aligned}$$

It suffices to show that for any $h \geq 0$ with $K(h) \geq 1$, $F(q_0, 0, h) > 0$ and $F(q_0, 1, h) \leq 0$.

To show that $F(q_0, 0, h) > 0$ for any $h \geq 0$ with $K(h) \geq 1$, it suffices to consider only $h < \bar{\mu} - 1$; otherwise, $I(h) = \emptyset$ and $K(h) = 0$. For h with $K(h) \geq 1$,

$$\begin{aligned} F(q_0, 0, h) & = K(h)[(1+r)(1-\beta_0) - (1+h)\beta]/\alpha + q_0 L(1-\beta_0)/\alpha - nM/\alpha \\ & > [(1+r)(1-\beta_0) - \bar{\mu}\beta]/\alpha + q_0 L(1-\beta_0)/\alpha - nM/\alpha \\ & \geq [(1+r)(1-\beta_0) - (\bar{\mu}\beta + nM)]/\alpha, \end{aligned}$$

which is non-negative under assumption SBAFD2. Thus, $F(q_0, 0, h) > 0$ for any h with $K(h) \geq 1$.

Now consider

$$\begin{aligned} F(q_0, 1, h) & = K(h)[(1+r)(1-\beta_0) - (1+h)\beta]/\alpha + q_0 L(1-\beta_0)/\alpha - nM/\alpha - \underline{R} \\ & < K(h)(1+r)(1-\beta_0)/\alpha + q_0 L(1-\beta_0)/\alpha - nM/\alpha - \underline{R} \\ & \leq n(1+r)(1-\beta_0)/\alpha + L(1-\beta_0)/\alpha - nM/\alpha - \underline{R} \\ & = n[(1+r) - M/\alpha] + L(1-\beta_0)/\alpha - \underline{R}. \end{aligned}$$

To see that this is negative, note that $\alpha = (1-\beta-\beta_0) < (1-\beta)$ implies $(1+r) - M/(1-\beta) > (1+r) - M/\alpha$, which is positive under assumption SBAFD2. Additionally, by the assumption,

$$\begin{aligned} n &\leq (\underline{R} - L/\alpha)/[(1+r) - M/(1-\beta)] \\ &< (\underline{R} - L/\alpha)/[(1+r) - M/\alpha] \\ &< [\underline{R} - L(1-\beta_0)/\alpha]/[(1+r) - M/\alpha]. \end{aligned}$$

Therefore, $F(q_0, 1, h) < 0$ for any $h \geq 0$. ■

PROOF OF PROPOSITION 6: Recall that the CEO's objective is to maximize her bonus, which is maximized when the value created from project investment is maximized, i.e.,

$$\text{Max}_h \sum_{i \in I(h)} \mu_i - C(K(h)),$$

where $K(h) \equiv |I(h)|$ and $I(h) \equiv \{i \mid \mu_i \geq (1+h) \text{ for } \mu_i \text{ of } \boldsymbol{\mu}\}$.

Let $\omega(a)$ denote the index of the a th highest μ_i of $\boldsymbol{\mu}$, and hence $\mu_{\omega(a)}$ is the a th highest μ_i .¹¹ Define

$$A \equiv \max\{a \mid \mu_{\omega(a)} \geq C'(a) \text{ for } a \leq n\}.$$

It is straightforward to show that $C(k) \equiv (1+r)k + Lq^k$ is strictly convex with $C'(k) = (1+r) + Lq^k \ln(q) > 0$ for $k > k^* \equiv \ln[-L \ln(q)/(1+r)]/[-\ln(q)]$. The strict convexity of $C(k)$ means $C'(k)$ is increasing in k . So borrowing a loan amount beyond A to finance some project with $\mu_{\omega(a)} < \mu_{\omega(A)}$ can only reduce the value created from project investment. Because $K(h) = A$ for $h = C'(A) - 1$, setting the hurdle rate at this level is optimal to the CEO if any loan amount $a < A$ is suboptimal.

Note that the capital "supply curve" $C(k)/k = (1+r) + Lq^k/k$ is downward sloping. If the loan amount is reduced to $a < A$, the "average cost" of borrowing $(1+c_0) = (1+r) + Lq^a/a$ is higher for the projects financed while the number of projects financed is smaller. With the same $\boldsymbol{\mu}$ given, the value created from project investment must be lower if a loan amount $a < A$ is borrowed. ■

¹¹ For simplicity, ties are ignored; they are zero-probability events if $\boldsymbol{\mu}$ is drawn from a continuous probability distribution.

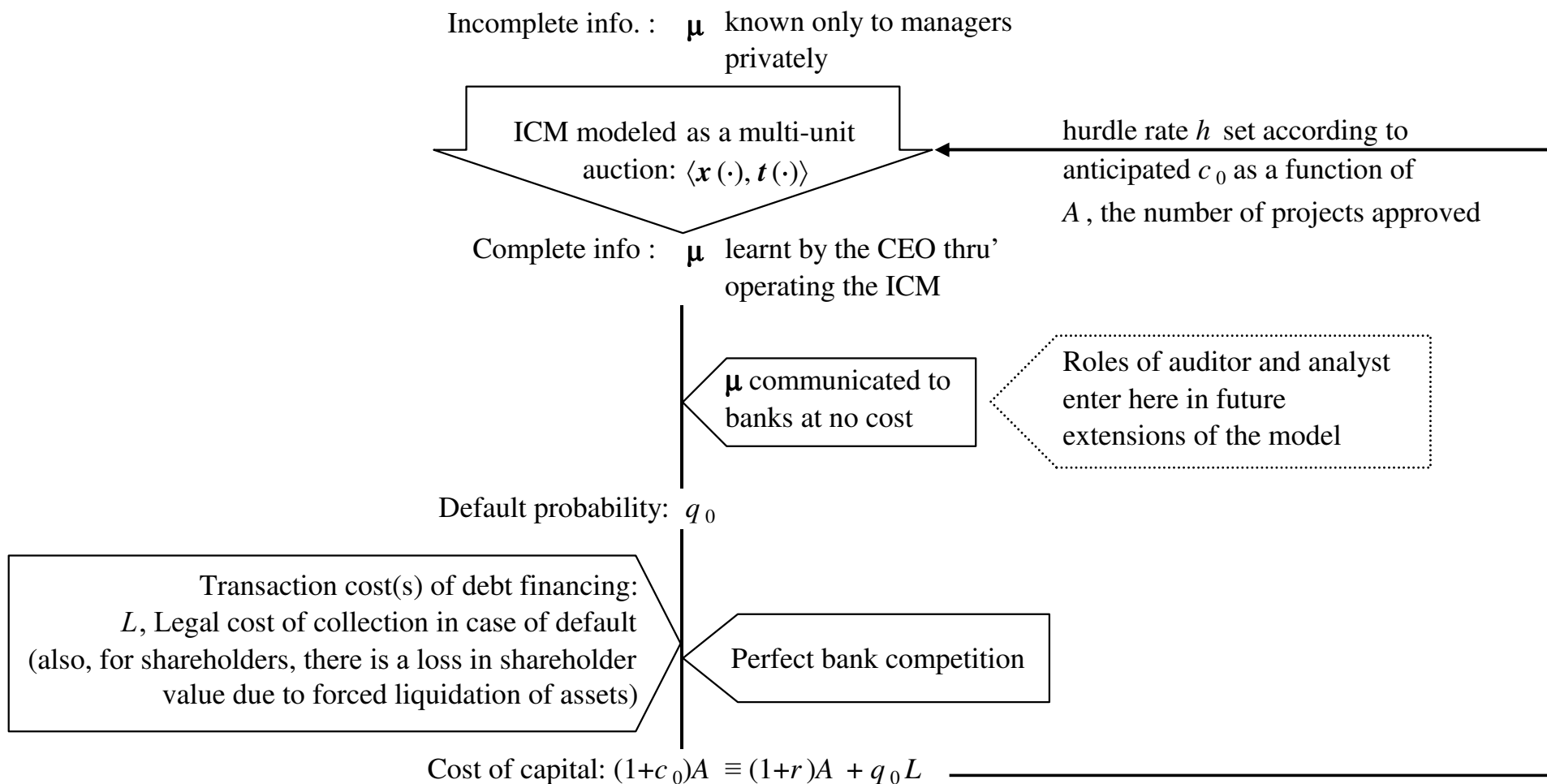


Figure 1. Important Elements of the Model

Figure 2. Geometric Default Probability Implies Capital Sponsoring and Subsidized Hurdle Rate

