

# EMPTY VOTING AND EFFICIENCY

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**ABSTRACT.** We study how the possibility of separating voting interests from economic ownership (“empty voting”) affects the efficiency of corporate governance. In our model, a strategic trader (such as a hedge fund) observes that a management proposal is up for a vote, and can accumulate a position in the stock prior to the record date. It can also effectively “buy” additional votes and thus hold more voting power than implied by its economic ownership, for example by borrowing shares or trading in derivatives markets. Between the record date and the voting date, the trader becomes informed about the value of the proposal and can again trade shares, this time in a market characterized by noise trading. On the voting date, the voting of atomistic stockholders is effectively randomly, while the strategic trader votes according to its economic incentives. We find that when the strategic trader holds no shares ex ante, its presence generally helps efficiency in terms of increasing the probability that the firm makes the right decision. We also find that increasing market depth and an increasing ability to capture votes separately from shares magnifies the positive effect. This occurs despite the fact that the trader will sometimes sell short after the record date and vote to decrease firm value. However, if the strategic trader holds a long position in the stock ex ante, an increase in market depth or increased ability to separate votes from ownership can reduce efficiency by inducing self-interested manipulation that would not occur otherwise.

**Keywords:** voting, trading, hedge funds, corporate governance

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## 1. INTRODUCTION

The impact of hedge funds on corporate governance has received considerable attention recently. The rise in popularity of hedge funds as investment vehicles has coincided with an increased focus on governance in general. Much of the attention has been focused on “activist” funds that take significant stakes in firms and then advocate for change (Brav, Jiang, Portnoy, and Thomas (2007)). However, certain strategies undertaken by hedge funds (or other strategic traders) can significantly affect the efficiency of corporate governance in more subtle ways.

In particular, recent work has shown that funds may use “empty voting” – a practice whereby they accumulate voting power in excess of their effective share ownership – to manipulate shareholder vote outcomes and generate trading gains. This practice is possible even when one share, one vote is the explicit rule. It can be accomplished, for example, by borrowing shares of stock on the record date or hedging economic exposure in the derivatives markets (see, e.g., Christoffersen, Geczy, Musto, and Reed (2007) and Hu and Black (2006a, 2006b)). Hu and Black (2006a, 2006b) provide a number of anecdotal examples where such behavior leads to perverse voting incentives. In one case, a hedge fund acquired votes through the share lending market, then voted against a buyout proposal and apparently profited from a short position when the share price dropped following the vote.<sup>1</sup> Hu and Black (2006a, 2006b) argue for regulation, including additional disclosure, to curb such activities.

Regulators have also expressed significant concern over empty voting, particularly given the recent boom in the hedge fund industry and the increasing number and importance of items requiring a shareholder vote. The Wall Street Journal (January 26, 2007, p. 1) quotes SEC chairman Christopher Cox as saying that the practice of empty voting “is almost certainly going to force further regulatory response to ensure that investors’ interests are

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<sup>1</sup>This incident involved a Hong Kong company named Henderson Land, which wanted to buy out a 25% minority interest in its publicly traded affiliate Henderson Investment.

protected...This is already a serious issue and it is showing all signs of growing.” Many large institutional shareholders are examining their share lending practices as well.

On the other hand, Christoffersen, Geczy, Musto, and Reed (2007) argue that “vote trading” in the share lending market can increase efficiency because information about proposals can be costly to acquire. Uninformed shareholders who are not willing to pay the cost to become informed can sell their votes to informed parties in order to increase the efficiency of the voting outcome. Of course, this argument requires that the vote buyer and vote seller have coincident interests, which seems to be violated in the examples cited by Hu and Black (2006a, 2006b). To date, there is no agreement on whether empty voting constitutes a significant problem that should be regulated. Importantly, the literature does not currently provide an integrated theoretical framework to help assess the tradeoff between increased information efficiency and the cost of possible manipulations via empty voting.

In this paper, we develop a theoretical model that explores this trade-off. We derive the optimal share and vote position of a strategic trader that has the ability to acquire information about the value of a proposal, and the ability to generate an empty voting position and probabilistically affect the outcome of the final vote. We show that while the trader may sometimes reduce efficiency by shorting the stock and then “voting the wrong way” (from a firm value perspective), the cost of these possible manipulations is often offset by a greater probability that the trader will “do the right thing” and vote to increase firm value. In other words, in equilibrium both the presence of the strategic trader *and* the ability to separate votes from economic ownership can increase overall efficiency by making the “right” outcome more likely. However, we also find situations where this logic is overturned.

Our analysis deals with deviations from the one share, one vote rule, on which there is a large existing literature dating back to at least Manne (1964). Much of the modern literature focuses on how the one share, one vote rule affects the efficiency of the market for corporate control (see, e.g., Harris and Raviv (1988), Grossman and Hart (1988), and Burkart and Lee (2007)), or how deviations between cash flow and voting rights held by *insiders* affects efficiency (e.g., DeAngelo and DeAngelo (1985), Gilson (1987)). These studies generally

focus on long-term deviations from one share, one vote that are codified in the corporate charter. In this study we focus instead on short-term deviations arising from activities in the derivatives or share lending markets, and how these deviations can affect the day-to-day efficiency consequences of voting by *outsiders*. We think of outsiders as parties who do not make proposals themselves, but face uncertainty over whether an insider's proposal is value-increasing or instead self-serving. There are many types of proposals other than proxy contests for board control that can have important value implications for the firm. Examples include proposals for the purchase of another firm, a divestiture, or a change in the corporate charter (often involving a takeover defense).

In our model, the firm's management proposes an action that requires shareholder approval. All shares are held either by atomistic shareholders or a single strategic trader. In the benchmark model, the strategic trader holds no shares prior to the announcement of the forthcoming vote. After the proposal is announced, the strategic trader can buy shares in a transparent market prior to the record date (ie, with no noise trading), and may also be able to separate its voting ownership from its economic ownership by paying a convex cost. This cost represents, for example, increasing difficulty in finding shareholders from whom to borrow shares, or the increasing cost of finding counterparties to hedge a large economic interest.

On the record date, voting interests are set according to share or vote ownership on that day - all votes are held by either atomistic shareholders or the strategic trader. After the record date, there is a significant time lag before the actual date of the vote, so the strategic trader is able to both learn about the value of the proposal, and further adjust its economic ownership (but not its voting interest). At this intermediate trading stage, however, the market is not completely transparent, because there may be noise trading by atomistic investors. Finally, on the voting date the strategic trader votes according to its economic incentives, while the voting of atomistic shareholders is effectively random. We do not explicitly model the atomistic holders' voting decisions; the important feature is that their behavior induces randomness in the final voting outcome.

Our assumptions are meant to reflect the realities of corporate governance in the United States. Christoffersen, Geczy, Musto, and Reed (2007) report that there is very little information known by the record date about the items up for a vote, and that there is a significant time gap between the record date and the meeting date (a median of 54 calendar days in their sample). Thus, it seems reasonable that there would be little ability to trade strategically from an informed position prior to the record date (which corresponds to our assumption of a transparent market at that stage), but a significant opportunity to gather information and trade less transparently between the record and voting dates.

It is important to note that we highlight two ways in which empty voting can occur in the U.S. corporate governance system. In addition to the lending and derivatives markets, there is also the time lag between record and voting dates. Even if voting and economic interests have to be aligned on the record date, it is possible to divorce the two prior to the voting date during the intervening time period. Our model allows us to separate the two effects.

We first solve a benchmark model with no ex ante ownership by the strategic trader. In this case, the strategic trader optimally trades to a long position prior to the record date in order to set up the possibility of future trading gains. However, there is a tradeoff. On the negative side, greater economic ownership reduces trading gains for two reasons. First, the “future self” of the trader will be concerned with protecting the value of its stake in addition to maximizing trading gains. This commitment effect of owning a significant stake on the record date thus reduces expected future trading profits. Second, the strategic trader’s position reduces overall market depth. On the positive side, a larger stake increases the trader’s voting power and thus its ability to affect the voting outcome. In equilibrium, the extent of the long position is determined by the expected amount of noise trading between the record and voting dates, and the ease with which votes can be acquired separately from shares. For the bulk of the analysis, we assume that the cost of separating votes from ownership is high enough that the trader will not acquire enough votes to singlehandedly determine the outcome.

After the record date, the strategic trader becomes informed about whether the proposal is good or bad and then plays a mixed strategy; it either buys additional shares and votes to increase firm value, or it sells to a short position and votes to decrease firm value. Given the effectively random voting of the atomistic shareholders, we find that the presence of the strategic trader is good for efficiency overall. Because of its long position on the record date, it tends to “vote the right way” more often than not, increasing the probability of a correct decision. As market depth increases, the optimal long position on the record date increases, intensifying the positive effect. Thus, we find that allowing for trading gains by a strategic trader can increase efficiency even though the trader sometimes engages in value-reducing strategies. Also, since the strategic trader would not acquire any votes in the absence of possible trading gains, noise trading improves voting efficiency.

We also find that the possibility of acquiring separate voting and economic stakes on the record date tends to decrease the trader’s economic ownership (in order to maximize trading gains – as noted above). This increases the probability that the trader will go short and vote the wrong way later. However, the net effect on efficiency is generally positive when votes are not too cheap, since the acquired votes also increase the trader’s ability to affect the voting outcome. This increased leverage outweighs the higher probability of voting the wrong way. This result can be overcome, however, if votes are so cheap that the trader can acquire enough to determine the vote outcome singlehandedly. In this case, the trader holds a zero economic position on the record date, and the probability of a correct decision can decrease.

Finally, we analyze how the model would change if the strategic trader had an ex ante long position in the stock. We find that the trader’s final economic interest on the record date is increasing in its initial ownership. If initial ownership is high enough, the trader would actually find it optimal to forgo trading gains, and instead buy enough shares and/or votes to ensure that the vote always goes the right way. In such a case, an increase in market depth or a decrease in the price of votes separated from shares can reverse that decision, and cause the trader to manipulate in order to generate trading gains, reducing overall efficiency.

Our results may provide some guidance on the efficacy of proposed regulatory reforms designed to curb or eliminate empty voting. For example, Hu and Black (2006a, 2006b) advocate additional disclosure requirements as a reasonable starting point. In the framework of our model, disclosure of an “empty voting” position on the record date would have no effect, because we already assume that the market maker observes the strategic trader’s actions at that stage. Disclosure of a change in economic position relative to voting rights between the record and voting dates would have the effect of reducing or eliminating any trading profits the strategic trader could otherwise generate. In our model, this would reduce the trader’s incentive to accumulate shares on the record date unless it were already a significant shareholder. In our benchmark model, such a rule would tend to reduce overall efficiency. In the model with initial holdings by the strategic trader, it would sometimes increase efficiency by convincing the trader to accumulate votes solely to maximize firm value.

**1.1. Related Literature.** Our model obviously involves a form of stock price manipulation, but where the manipulation is accomplished by affecting the firm’s physical operations. Closely related are Kyle and Vila (1991), Maug (1998), and Kahn and Winton (1998). In all of these models, a strategic trader can directly take an action that will affect firm value, and its ability to trade in a noisy stock market affects its incentives to do so. The main difference between our analysis and theirs is that we endogenize the trader’s ability to affect firm value by modeling the voting game and deriving the optimal ex ante share and vote position of the trader. In a related vein, Goldstein and Guembel (2007) study a situation where a probabilistically informed speculator can affect the firm’s real investment decisions indirectly, by moving stock prices. In their model, managers try to infer the speculator’s information by observing price movements. The authors find that when the speculator turns out to be uninformed, it will sometimes manipulate the price downward by selling short, which can be profitable for two reasons. First, because the stock is already priced too high (given the speculator’s knowledge that he is uninformed), and second, because the manipulation

may cause management to forego profitable investments after the observed price decline. Other models of manipulation involving real activities include Bagnoli and Lipman (1996) and Vila (1989), both of which study manipulation involving actions such as a takeover bid. Manipulation utilizing information alone has also been studied widely, such as by Allen and Gale (1992) and Chakraborty and Yilmaz (2004).

Brav, Jiang, Portnoy, and Thomas (2007) study hedge fund activism, and show that funds target undervalued firms and often aggressively advocate for changes in firm policies. Such a strategy contrasts sharply with the trading strategy we study in this paper, where the fund reacts to a management proposal, and optimally keeps its information about the value of a proposal private in order to generate trading gains.

Our analysis is also closely related to the small but growing literature on vote buying. For example, Blair, Golbe, and Gerard (1989) and Neeman and Orosel (2007) show that allowing a contest for votes in addition to a contest for shares can have efficiency advantages. However, they do not model how stock trading interacts with vote buying.

Other authors have modeled trading and voting together, but without allowing for “empty voting.” For example, Maug (1999) models a strategic voting game in which voting and trading both help aggregate dispersed information about the value of a proposal. Musto and Yilmaz (2003) study how the operation of a financial market affects political voting.

Our study is also related to papers studying the value of corporate votes. For example, Zwiebel (1995) models shareholders’ incentives to form blocks and participate in voting coalitions. Barclay and Holderness (1989) study block trades and find that there is a significant premium paid for large minority blocks, which indicates that less than majority voting control can be valuable.

The paper proceeds as follows. In section 2 we describe the model. In section 3 we derive the equilibrium in the absence of initial shareholdings. In section 4 we allow the strategic trader to hold shares *ex ante*. We conclude in section 5.

## 2. THE MODEL

The model focuses on a firm with an upcoming shareholder vote. The firm has one perfectly divisible share outstanding. The players consist of the management of the firm, a strategic trader, a market maker, and atomistic shareholders. Management sets the agenda for the vote, but does not hold any shares and cannot vote on its own account. The market maker also does not vote.

At the beginning of the game, management makes a proposal. The proposal can be either good or bad. If the proposal is defeated, firm value is  $v$ , which is common knowledge. If the proposal is approved, firm value increases by  $\Delta v$  to  $\bar{v}$  if it is a good proposal, and decreases by  $\Delta v$  to  $\underline{v}$  if it is a bad proposal. We do not model the reason why management may make a bad proposal. As an example, it could be caused by an agency problem or a lack of ability or information on management's part.

All players are initially uninformed about whether the proposal is good or bad. The strategic trader, hereafter H, may hold an ex ante long position in the firm's stock equal to  $\alpha_h$  shares (or, equivalently,  $\alpha_h$  percent of the shares). After the proposal is announced, but before the record date, H can submit a market order to buy shares in the firm. The order is filled by the market maker at a price equal to the expected value of the shares (ie, the market maker and H have the same information at this point, including any "extra" votes H will acquire, and H's trade is transparent to the market maker). For simplicity, we assume that the market maker holds no inventory. So, if it sells shares to H, it is able to immediately purchase those shares from atomistic holders. This assumption simplifies the analysis because it implies that all of the shares will be held by either H or atomistic stockholders on the record date. The important feature of the assumption is that ownership of shares by H reduces ownership by atomistic shareholders, who are the only other parties allowed to vote, and who also determine later market liquidity. We denote H's final long position on the record date by  $\alpha_H$ . We do not allow H to short the stock prior to the record

date. It is easy to show that shorting would never be optimal unless H can actually increase market liquidity by shorting.

The strategic trader may also be able to acquire votes in addition to those represented by its record date share ownership. It can do this at a cost,  $c(\alpha_X)$ , that is increasing and convex in the number of “extra” votes,  $\alpha_X$ . This cost reflects any expense H incurs in separating its voting interest from its economic ownership. For example, the extra votes could be purchased on the share lending market. When H approaches a given atomistic shareholder to borrow its shares, H may have all of the bargaining power, and thus be able to borrow the share at effectively zero cost – the shareholder does not believe its vote will be pivotal, and thus is willing to sell the vote for any nominal cost. Note that Christoffersen, Geczy, Musto, and Reed (2007) find that the average vote sells for zero in the share lending market. However, since the lending market is decentralized H must first find the shareholder. Our assumption then corresponds to a convex search cost function (it becomes harder to find the next shareholder the more you have already located).

On the record date, all shares are held either by H or by atomistic shareholders. Their holdings on that date determine their final voting power. Following the record date, we assume that both H and the market maker costlessly become informed about whether the proposal is good or bad (note that it is possible to construct an alternative model with more simplistic trading strategies in which the market maker does not become informed – the qualitative results of that analysis are similar). The atomistic shareholders do not become informed. Next, some atomistic shareholders are hit by a random liquidity shock that causes them to sell their interest in the firm. We assume that with probability  $\frac{1}{2}$ , a proportion  $\alpha_Z$  of the atomistic shareholders place market orders to sell their shares, and that otherwise there is no trading by atomistic shareholders. Note that since H owns  $\alpha_H$  shares at this point, the total number of shares sold by atomistic holders when the liquidity shock hits is  $\alpha_Z(1 - \alpha_H)$ . H can also place a market order at this time to buy or sell whatever quantity it wishes (without first observing whether atomistic shareholders sell). The market maker observes only the total net order flow. There are no short sales constraints.

Finally, on the voting date, H votes its  $\alpha_H + \alpha_X$  votes according to its own economic incentives and information. For the  $1 - (\alpha_H + \alpha_X)$  votes held by atomistic stockholders, we assume that the *proportion* of “yes” votes cast, denoted by  $Y$ , is uniformly distributed on  $[0,1]$ . Note that the atomistic holders’ total number of “yes” votes therefore equals  $Y(1 - \alpha_H - \alpha_X)$ . If at least  $\frac{1}{2}$  of the total votes are cast in favor, the proposal passes. The resulting value of the firm is then realized and immediately reflected in the share price.

Note that we assume the atomistic shareholders remain uninformed about whether the proposal is good (as reflected in their voting behavior), whereas the market maker becomes informed. This implies that the atomistic shareholders could become informed simply by observing the market price, but fail to do so. We believe this assumption captures some important features of the real world. Many shareholders own shares in a given firm for diversification reasons only, and would face a significant cost to become informed about a given proposal. Even if the information could be learned from the market price, many shareholders lack the expertise required to decode the information contained in prices. In any case, what is important for our analysis is that there is some randomness in the final vote outcome that can be exploited by the strategic trader, which certainly seems to be true of many situations in the real world. Our assumptions amount to a simple way of capturing this in a tractable model.

Figure 1 illustrates the timeline of the game.

### 3. EQUILIBRIUM WITH NO EX ANTE HOLDINGS

We first solve the model assuming no ex ante shareholdings by H ( $\alpha_h = 0$ ). We derive a subgame perfect Nash equilibrium using backward induction. For ease of exposition, we assume until otherwise noted that the proposal is good (i.e., value increasing). We also assume that  $\alpha_Z < \frac{1}{2}$  and that the cost function  $c(\alpha_X)$  is such that  $\alpha_X + \alpha_H < \frac{1}{2}$  in equilibrium



simultaneously by H. We first derive the equilibrium assuming this behavior, then prove that it is the unique equilibrium of the voting game in Lemma 1 below.

In order for the mixed trading strategy to be profitable, H must set its buy and sell quantities such that the market maker cannot always see what H is doing. In particular, the market maker must be unable to distinguish between cases where atomistic holders sell while H buys, and cases where H sells while atomistic holders do not sell. This provides the following constraint on H's buy/sell quantities:

$$(1) \quad \alpha_Z(1 - \alpha_H) - \alpha_B = \alpha_S.$$

We must now determine the optimal mixing probability,  $q$ , and optimal buying quantity,  $\alpha_B$ . Let  $V_B$  denote the expected value of the firm given that H buys  $\alpha_B$  additional shares and votes in favor of the proposal. Then we must have

$$V_B = \bar{v}Pr \left[ Y(1 - \alpha_H - \alpha_X) + \alpha_H + \alpha_X \geq \frac{1}{2} \right] + vPr \left[ Y(1 - \alpha_H - \alpha_X) + \alpha_H + \alpha_X < \frac{1}{2} \right].$$

Using the properties of the uniform distribution, this becomes

$$V_B = \bar{v} \left( 1 - \frac{\frac{1}{2} - \alpha_H - \alpha_X}{1 - \alpha_H - \alpha_X} \right) + v \left( \frac{\frac{1}{2} - \alpha_H - \alpha_X}{1 - \alpha_H - \alpha_X} \right).$$

We define  $V_S$  similarly as the expected value of the firm conditional on H selling and voting against the proposal. This equals

$$V_S = \bar{v}Pr \left[ Y(1 - \alpha_H - \alpha_X) \geq \frac{1}{2} \right] + vPr \left[ Y(1 - \alpha_H - \alpha_X) < \frac{1}{2} \right].$$

Using the properties of the uniform distribution, this becomes

$$V_S = \bar{v} \left( 1 - \frac{\frac{1}{2}}{1 - \alpha_H - \alpha_X} \right) + v \left( \frac{\frac{1}{2}}{1 - \alpha_H - \alpha_X} \right).$$

Intuitively, H voting against the proposal just creates a higher hurdle for the atomistic holders to approve the proposal.

H's expected surplus from buying  $\alpha_B$  shares and voting in favor is

$$(2) \quad \alpha_B \left( V_B - \left[ \frac{1}{2}V_B + \frac{1}{2}[qV_S + (1 - q)V_B] \right] \right) + \alpha_H V_B.$$

The first term in the equation equals H's expected trading profits. The expected price at which H buys shares,  $[\frac{1}{2}V_B + \frac{1}{2}[qV_S + (1-q)V_B]]$  reflects the fact that half of the time, no atomistic holders will sell, and its strategy will be transparent to the market maker. The rest of the time it successfully hides among the noise, and the market maker sets the price at the "uninformed" expected value (i.e., it reflects the true value of the proposal, but not knowledge of whether H will vote for or against). The second term in the equation reflects the value of H's existing stake in the firm.

Similarly, H's expected profits from selling and voting against equals

$$(3) \quad (\alpha_B - \alpha_Z(1 - \alpha_H)) \left( V_S - \left[ \frac{1}{2}V_S + \frac{1}{2}[qV_S + (1-q)V_B] \right] \right) + \alpha_H V_S,$$

where the  $(\alpha_B - \alpha_Z(1 - \alpha_H))$  in the first term reflects the constraint given by equation (1). Equation (2) must equal (3) in order for H to be indifferent and willing to mix. In order to solve for the optimal  $q$  and  $\alpha_B$ , we set the two equations equal and first solve for the  $q$  that makes H indifferent for a given  $\alpha_B$ , and then maximize H's overall expected profits to solve for the optimal  $\alpha_B$  (we show in Lemma 1 that this is the unique equilibrium).

Setting (2) equal to (3) and solving for  $q$  yields

$$q = 1 - \frac{\alpha_B + 2\alpha_H}{\alpha_Z(1 - \alpha_H)}.$$

H's overall expected profit equals  $(1 - q)$  times (2) plus  $q$  times (3). Plugging the expression above for  $q$  into this overall profit function yields a concave function in  $\alpha_B$  that is easily shown to have a maximum at

$$\alpha_B^* = \frac{\alpha_Z}{2} - \frac{\alpha_H(2 + \alpha_Z)}{2}.$$

Plugging this into the expression for  $q$  above implies an optimal mixing quantity of

$$q^* = \frac{1}{2} - \frac{\alpha_H}{\alpha_Z(1 - \alpha_H)}.$$

We have the following result (all proofs are provided in the appendix).

*Lemma 1.* In the unique subgame perfect Nash equilibrium of the trading game, H plays a mixed strategy in which it sells  $\alpha_Z(1 - \alpha_H) - \alpha_B^*$  shares with probability  $q^*$  and buys  $\alpha_B^*$  shares with probability  $(1 - q^*)$ .

Using these optimal quantities, we can rewrite H's overall expected payoff for this stage as

$$\frac{(V_B - V_S)(\alpha_Z^2(1 - \alpha_H)^2 - 4\alpha_H^2)}{8\alpha_Z(1 - \alpha_H)} + \alpha_H(q^*V_S + (1 - q^*)V_B).$$

The first term represents H's expected trading profits, while the second is simply the expected value of its stake.

Most of the above analysis applies directly to the case of a bad proposal as well, given the symmetry of the problem. The only changes are in the expressions for  $V_B$  and  $V_S$ . Also, it is worth noting that the size of the "value wedge,"  $V_B - V_S$ , that H can generate by randomizing its vote is the same whether the proposal is good or bad, and equals  $\frac{\Delta v(\alpha_H + \alpha_X)}{1 - \alpha_H - \alpha_X}$ .

Given this optimal trading strategy, we can proceed to solve for H's optimal share and vote trading prior to the record date. First recall that H does not yet know at this stage whether the proposal is good or bad. H's expected profits viewed from this stage of the game include its expected future trading profits plus the expected value of the stake it acquires today, less the price it pays for the stake and the cost of any "excess" votes,  $c(\alpha_X)$ . Since good and bad proposals are symmetric, and the value wedge that H can generate is the same in each case, H's expected future trading profits are always the same as derived above for the case of a good proposal, or  $\frac{(V_B - V_S)(\alpha_Z^2(1 - \alpha_H)^2 - 4\alpha_H^2)}{8\alpha_Z(1 - \alpha_H)}$ . Given our assumption that the shares H buys at this stage will be priced at their true expected value, the price of the stake will exactly offset its expected value, so H will choose its stake solely to maximize expected future trading profits. We thus have the objective function

$$(4) \quad \max_{\alpha_H, \alpha_X} \frac{(V_B - V_S)(\alpha_Z^2(1 - \alpha_H)^2 - 4\alpha_H^2)}{8\alpha_Z(1 - \alpha_H)} - c(\alpha_X),$$

or, equivalently (from above)

$$(5) \quad \max_{\alpha_H, \alpha_X} \frac{\left( \frac{\Delta v(\alpha_H + \alpha_X)}{1 - \alpha_H - \alpha_X} \right) (\alpha_Z^2 (1 - \alpha_H)^2 - 4\alpha_H^2)}{8\alpha_Z(1 - \alpha_H)} - c(\alpha_X).$$

The basic tension in H's stake purchase decision can be seen in these equations. For a given value wedge ( $V_B - V_S$ ), trading profits are maximized by choosing a stake of  $\alpha_H = 0$ . This is reflected in the term  $\frac{(\alpha_Z^2(1-\alpha_H)^2 - 4\alpha_H^2)}{8\alpha_Z(1-\alpha_H)}$ , which is decreasing in  $\alpha_H$ . Intuitively, the larger the stake H owns at the later trading stage, the more it will worry at that point about protecting the value of that stake rather than generating trading gains. Furthermore, H's ownership offsets ownership by atomistic shareholders, and thus reduces market depth and the potential for profitable trading.<sup>2</sup> On the other hand, the greater the stake, the more voting power H has, so the larger is the value wedge it can create. This is reflected in the term  $\frac{\Delta v(\alpha_H + \alpha_X)}{1 - \alpha_H - \alpha_X}$ . The optimal stake trades off these two effects. The optimal amount of extra votes,  $\alpha_X$ , affects trading profits only indirectly, through the increased value wedge. Thus, this positive effect is weighed against the direct cost  $c(\alpha_X)$ .

Analyzing (5) under our assumption that  $c(\alpha_X)$  is such that H will not take complete control of the voting decision yields the following result.

*Proposition 2.* The solution to (5) is characterized by a unique optimal equity stake,  $0 < \alpha_H^* < \frac{\alpha_Z}{2 + \alpha_Z}$ , and optimal voting stake  $\alpha_X^*$ .

H's optimal record date equity stake is clearly limited by the ability to generate trading gains. If the stake gets too large (larger than  $\frac{\alpha_Z}{2 + \alpha_Z}$ ), the desire to protect the value of the stake will overcome any incentive to profit by trading. Thus, when choosing an ex ante stake purchased at its expected value, H will significantly limit the size of the stake purchase.

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<sup>2</sup>Note that if this objective function were valid for a short position, ie  $\alpha_H < 0$ , it would actually imply that a short position is optimal to maximize trading gains. However, this is an artifact of the fact that the equation is constructed assuming market depth equals  $\alpha_Z(1 - \alpha_H)$ , which would imply that H can increase market depth by going short. We do not think this is reasonable, so we have chosen to rule out a short position on the record date

However, the stake is always positive since share ownership increases H's ability to affect the vote and create a value wedge.<sup>3</sup> The size of H's "extra" vote stake,  $\alpha_X$ , is simply driven by the tradeoff between voting influence and the cost of the votes.

Now that we have the solution to H's share and vote purchase strategy, it remains to determine how H's actions will affect efficiency overall. We measure efficiency by the probability with which the correct decision is made by the voting shareholders. For now, assume the proposal turns out to be good. In this case, given that H votes against the proposal with probability  $q$ , the ex ante probability of approval is

$$(6) \quad q \left( Pr \left[ Y(1 - \alpha_H - \alpha_X) > \frac{1}{2} \right] \right) + (1 - q) \left( Pr \left[ Y(1 - \alpha_H - \alpha_X) + \alpha_H + \alpha_X > \frac{1}{2} \right] \right).$$

Plugging in the equilibrium probability  $q^*$  from above and using the properties of the uniform distribution for  $Y$ , this becomes

$$(7) \quad \frac{1}{2} + \frac{\alpha_H}{\alpha_Z(1 - \alpha_H)} \left( \frac{\alpha_H + \alpha_X}{1 - \alpha_H - \alpha_X} \right).$$

It is easy to show that the probability of a correct decision in the case of a bad proposal is exactly the same (since the decision threshold is  $\frac{1}{2}$ ). Since the probability of the correct decision without H is  $\frac{1}{2}$  given the uniform distribution for  $Y$ , the following result is immediate.

*Proposition 3.* The presence of the strategic trader increases the ex ante probability of a correct decision.

Thus, despite the fact that H will seek to generate trading profits by sometimes voting the wrong way and manipulating the firms' decisions to decrease value, its presence overall is actually beneficial to the firm from an ex ante perspective. This is because the positive share position H takes in order to increase voting power and create a value wedge causes it to vote the right way more often than not. Thus, the more rare cases where H manipulates

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<sup>3</sup>Note that given our assumption that  $\alpha_Z < \frac{1}{2}$  and the fact that trading profits will go away if  $\alpha_H \geq \frac{\alpha_Z}{2 + \alpha_Z}$ , to justify our maintained assumption on  $c(\alpha_X)$  it is sufficient to assume that the cost function is "steep enough" that  $\alpha_X < \frac{1}{4}$  in equilibrium.

negatively can be seen as the “price to be paid” for greater overall voting efficiency. Note that if there were no possibility of trading gains ( $\alpha_Z = 0$ ), H would have no incentive to purchase shares or votes, or to try to learn the value of the proposal. It is the possibility of trading gains that induces the information gathering and therefore increases efficiency.

It is worth noting at this point that these results will clearly be somewhat sensitive to our assumptions on the acceptance threshold and the distribution of votes by atomistic shareholders. The key to our basic results is that the “noise” induced by the random votes is centered around the threshold. In other words, the expected proportion of yes votes by atomistic stockholders is the same as the acceptance threshold,  $\frac{1}{2}$ . Clearly, if that were not the case, the outcome of the vote would be more predictable, and H’s ability to create a value wedge and thus its incentives to acquire votes would be affected. In such cases, it is also possible for H’s presence to be bad for overall efficiency. For example, if  $Y$  is distributed uniformly, but acceptance of the proposal requires a significant supermajority, it is possible to show that the presence of H reduces the probability of accepting a good proposal, but decreases the probability of accepting a bad proposal. From an ex ante perspective, the net effect on efficiency is often ambiguous. Thus, one could view our result as a first step showing that strategic trading coupled with empty voting can increase overall efficiency in some cases.

We now investigate how changes in the exogenous parameters affect the equilibrium. The model’s tractability is somewhat limited at this stage, so we first derive analytical results by assuming that votes and shares cannot be separated on the record date, i.e.  $\alpha_X = 0$ .

*Proposition 4.* Assume  $\alpha_X = 0$ . Then the optimal stake  $\alpha_H^*$  is increasing in  $\alpha_Z$ , and is unaffected by  $\Delta v$ .

The greater is market depth ( $\alpha_Z$ ), the greater is the potential for trading profits, and the more equity H will purchase, in general, in order to take advantage. However, it is interesting to note that the importance of the proposal ( $\Delta v$ ) does not affect H’s stake purchase decision in the absence of a market for votes on the record date. The magnitude of  $\Delta v$  will certainly

affect the magnitude of trading profits H is able to generate, but it does not affect the trade-off between trading profits and the ability to generate a value wedge, which is what determines the optimal stake.

It is also interesting to consider how an increase in the depth of the market,  $\alpha_Z$ , affects overall efficiency. From (7), an increase in  $\alpha_Z$  seems to reduce the probability of the correct decision, but from Proposition 2 we know that an increase in  $\alpha_Z$  tends to increase  $\alpha_H$ . It is not tractable to analyze the net effect analytically, but numerical solutions show that the effect of an increase in  $\alpha_Z$  is unambiguously positive for efficiency.

Now consider how a change in the cost of buying votes separate from shares,  $c(\alpha_X)$ , affects equilibrium efficiency. This affects H's strategy only through its primary effect on  $\alpha_X$ , which then also affects  $\alpha_H$ . Again, the comparative statics are not analytically tractable, but numerical simulations assuming  $c(\alpha_X) = c\frac{\alpha_X^2}{2}$  for some  $c > 0$  show that a decrease in  $c$ , which increases  $\alpha_X$ , will decrease  $\alpha_H$  but increase overall efficiency, as long as the parameters are such that H does not take full voting control (as we have assumed). The decrease in  $\alpha_H$  occurs because H can achieve voting power by buying votes, which offsets the need to acquire shares in order to create a value wedge. This implies that H will vote the wrong way more often. However, the overall effect on efficiency is positive because H is still voting the right way more often than not, and the extra voting power it attains through the derivatives or lending markets offsets the decrease in the probability of voting the right way.

We have thus far maintained the assumption that buying votes separated from shares is sufficiently expensive that H will never be able to deterministically swing the election. If this assumption were relaxed the equilibrium would change. In the extreme, if separating votes from ownership at the record date were free ( $c(\alpha_X) = 0$ ), the equilibrium would be as described in the following result.

*Proposition 5.* Assume  $c(\alpha_X)=0$ . Then in equilibrium, H will not trade in the stock prior to the record date (i.e.,  $\alpha_H^* = 0$ ), but will accumulate sufficient votes to determine the election

outcome (i.e.,  $\alpha_X^* \geq \frac{1}{2}$ ). The probability of H selling short and voting to reduce firm value will be  $\frac{1}{2}$  in equilibrium, and H's presence will not affect ex ante efficiency.

This result reflects the fact that H's trading gains are maximized when its stake is zero. Since buying enough votes to swing the election maximizes the value wedge,  $V_B - V_S$ , there is no longer any reason for H to take a position in the stock on the record date. Then it maximizes its trading profits by buying/selling with equal probability, so while the *conditional* probability of the correct decision if H later sells short will certainly be lower, the *ex ante* probability of a correct decision is the same since H improves efficiency when it buys. It is interesting to note again that this is an extreme version of the model in which H's ability to manipulate firm value is maximized, yet the overall effect is at worst neutral for ex ante firm value.

#### 4. EX ANTE SHAREHOLDINGS

In this section, we consider how the equilibrium changes if the strategic trader holds a long position in the stock,  $\alpha_h > 0$ , prior to the announcement of a proposal. In other words, if H is already a long-term stockholder. For tractability, we assume for now that buying votes separately from shares at the record date is impossible, so  $\alpha_X = 0$ . To keep the notation consistent with the analysis above, we assume that  $\alpha_H$  represents H's final record date position, so his pre-record date trading quantity is  $\alpha_H - \alpha_h$ .

In this case, H will take into account the value of its existing stake when considering how to trade prior to the record date. However, following the record date the model is solved exactly as before, with one caveat. In some cases H will end up owning more than  $\frac{\alpha_Z}{2+\alpha_Z}$  shares, in which case H will always vote its shares the correct way (since manipulating to produce trading profits is not optimal - i.e. the optimal  $q$  is zero when  $\alpha_H \geq \frac{\alpha_Z}{2+\alpha_Z}$ ).

H's objective function for pre-record date trading, assuming  $\alpha_H < \frac{\alpha_Z}{2+\alpha_Z}$ , becomes

$$(8) \quad \max_{\alpha_H} \frac{\left(\frac{\Delta v(\alpha_H)}{1-\alpha_H}\right) (\alpha_Z^2(1-\alpha_H)^2 - 4\alpha_H^2)}{8\alpha_Z(1-\alpha_H)} + E[\alpha_h(q^*V_S^* + (1-q^*)V_B^*)]$$

where the expectation is taken over the probability of a good proposal versus a bad one. We use the superscript \* in the second term to denote the fact that the quantities are being evaluated at their equilibrium values (derived above) given  $\alpha_H$ . Intuitively, H is now simply maximizing its future trading gains, represented by the first term, but is also taking into account the effect of its future actions on the value of its initial stake, represented by the second term.

In the model with no ex ante ownership, it was never optimal for H to accumulate a stake as large as  $\frac{\alpha_Z}{2+\alpha_Z}$ , because in the future trading round it would then never have an incentive to manipulate and generate trading gains. However, when H has an ex ante position, this can change. It is easy to show that if H is always going to vote the right way, it is optimal for H to acquire enough shares to have full control over the vote outcome ( $\alpha_H^* \geq \frac{1}{2}$ ) and ensure the right decision. This is because the shares are bought at their expected “cash flow” value, but the value of the votes conferred on the holder is not reflected in the price. Since these votes will have positive value to H, it is willing to acquire the shares.

Using this, and analyzing (8) yields the following result.

*Proposition 6.* There exists a threshold level of initial holdings,  $\hat{\alpha}_h < \frac{\alpha_Z}{2+\alpha_Z}$ , such that:

- a) for all  $\alpha_h < \hat{\alpha}_h$ , H purchases additional shares to reach a record date position  $\alpha_h < \alpha_H^* < \frac{\alpha_Z}{2+\alpha_Z}$ ; and
- (b) for all  $\alpha_h \geq \hat{\alpha}_h$ , H trades to reach a record date position of  $\alpha_H = \frac{1}{2}$ , always votes to maximize firm value, and thus maximizes overall efficiency.

When H has initial shareholdings, it is willing to sacrifice some trading profits in order to protect the value of its existing stake, thus it favors a larger stake than in the benchmark model. If the initial stake is large enough, it gives up on future trading profits entirely, and makes sure that it has sufficient votes to guarantee the right voting outcome.

We can also derive the following comparative static for the extended model.

*Proposition 7.* The threshold level of initial holdings,  $\hat{\alpha}_h$ , is increasing in  $\alpha_Z$ .

Just as in the benchmark model, greater market depth induces greater trading gains, which means that a larger initial stake is required to induce H to give up those gains. Together with the prior result, we can now say something about how an increase in market depth may affect overall efficiency.

*Corollary 8.* Consider an increase in  $\alpha_Z$ . Then for all  $\alpha_h$  such that  $\alpha_h \geq \hat{\alpha}_h$  before the increase, but  $\alpha_h < \hat{\alpha}_h$  afterwards, the increase in market depth reduces ex ante efficiency.

Here we get a different result than in the benchmark model. Increased market depth can be bad for efficiency because it might convince a large shareholder who would otherwise have given up on trading gains (so as to maximize firm value) to instead try to generate some trading gains by threatening to short sell and vote the wrong way.

Finally, consider what happens if H is allowed to buy votes separately from shares on the record date. For tractability, consider the extreme case where  $c(\alpha_X) = 0$ . In this case, H can swing the election no matter its record date holding. Given our assumption that H does not trade to a short position, we have the following result.

*Proposition 9.* Assume  $c(\alpha_X) = 0$ . Then H will accumulate sufficient votes to determine the election outcome (i.e.,  $\alpha_H^* + \alpha_X^* \geq \frac{1}{2}$ ), and:

- (a) if  $\alpha_h \leq \frac{\alpha_Z^2}{8}$ , H sells to reach a position of  $\alpha_H^* = 0$ ; or
- (b) if  $\frac{\alpha_Z^2}{8} < \alpha_h < \frac{\alpha_Z}{2+\alpha_Z}$  H sells to reach a position  $0 < \alpha_H^* < \alpha_h$ ; or
- (c) if  $\alpha_h \geq \frac{\alpha_Z}{2+\alpha_Z}$ , H will not trade, but will always vote ex post to maximize firm value.

Comparing this result to Proposition 6, it is clear that allowing H to buy votes separate from economic ownership at the record date can be bad for efficiency. In Proposition 9, H ends up owning fewer than  $\frac{\alpha_Z}{2+\alpha_Z}$  shares anytime  $\alpha_h \leq \frac{\alpha_Z}{2+\alpha_Z}$ , so it short sells and votes to reduce firm value with some probability in the future. On the other hand, in Proposition 6, in the range of  $\alpha_h \in [\hat{\alpha}_h, \frac{\alpha_Z}{2+\alpha_Z}]$ , full efficiency is achieved since H finds it optimal to take voting control and maximize firm value. So for at least all  $\alpha_h$  in that range, allowing the separation of votes and ownership at the record date decreases efficiency.

## 5. CONCLUSION

We provide a model of empty voting in the U.S. corporate governance system. We find that allowing empty voting can actually improve overall efficiency despite the fact that it may cause a strategic trader to sometimes “vote the wrong way” in order to generate trading gains. In our benchmark model, making empty voting easier and increasing market depth both have positive effects on efficiency. However, when we consider the fact that the strategic trader could already be a long-term shareholder in the stock, we find that these results can be reversed. Increasing market depth or decreasing the costs of separating votes from economic ownership can decrease efficiency by causing the shareholder to use his influence to generate trading gains rather than ensure an efficient voting outcome. As discussed in the introduction, these results should be useful in the evaluation of possible regulatory changes, such as the augmented disclosure rules advocated by Hu and Black (2006a, 2006b).

While our model provides a coherent framework for addressing the efficiency consequences of empty voting, there are a number of issues that remain unexplored. For example, it would be interesting to study how the results would change if there were multiple strategic traders who compete to generate trading gains. It would also be interesting to study the interaction between a non-shareholder and an existing large shareholder who could both act to influence the vote outcome. Finally, we would like to more closely investigate specific mechanisms by which shares and votes can be separated, such as the share lending market. For example, if the uninformed shareholders were not all atomistic, how would they analyze the decision of whether to lend their shares on the record date? Our framework should provide a platform for exploring these issues in the future.

## APPENDIX

**Proof of Lemma 1:** Any realization of total net order flow, say  $n$ , (which is all the market maker observes) could occur in two possible ways: atomistic holders could have placed no orders while H placed an order of  $n$ , or atomistic holders could have placed an order to sell  $\alpha_Z(1 - \alpha_H)$  while H placed an order equal to  $n + \alpha_Z(1 - \alpha_H)$ . For all  $n$ , we call each pair of possible trading quantities for H,  $(n, n + \alpha_Z(1 - \alpha_H))$  a “quantity pair,” indexed by  $n$ .

In any subgame perfect equilibrium, upon observing net order flow of  $n$  the market maker must set a price that is optimal given H’s equilibrium (mixed) trading strategy. Now recall that if H ends up with a long position, it will always vote in favor of a good proposal, while if it ends up with a short position it will always vote against. Because of this, if H places any mixing weight on either element of a quantity pair such that H would end up long with either order or short with either order in the quantity pair (ie, quantity pairs such that  $n > -\alpha_H$  so that H would end up long with either quantity, or pairs such that  $n + \alpha_Z(1 - \alpha_H) < -\alpha_H$  so that H would end up short with either quantity), the optimal price for the associated order flow  $n$  must be either  $V_B$  or  $V_S$  since there is no uncertainty over H’s voting behavior. Thus, H cannot generate any trading gains by placing such an order, and no such order can be part of an equilibrium strategy for H.

This implies that H will only consider trading quantities that are elements of quantity pairs such that  $n + \alpha_Z(1 - \alpha_H) > -\alpha_H$  and  $n < -\alpha_H$  (hereafter “feasible pairs”), so that H would end up long after the former order and short after the latter. Next note that if H puts mixing weight on one element of a feasible pair, it must also put weight on the other element. If it did not, then in equilibrium the market maker upon observing the associated  $n$  would be able to infer H’s final trading position, and would price the stock accordingly, so that no trading gains are possible.

Next we prove that H puts weight only on the elements of one feasible pair, corresponding to the strategy in the lemma. First note that for any given feasible pair, there is a unique price corresponding to the associated  $n$  that will make H indifferent between the two quantities

in that pair. The prices will be different for different feasible pairs, and must correspond to the prices derived by plugging the mixing quantity  $q$  from equation (3) into the pricing equation  $qV_S + (1 - q)V_B$ . Thus, the equilibrium price  $p_n$  for a given  $n$  must satisfy  $p_n = (\frac{n+2\alpha_H}{\alpha_Z(1-\alpha_H)})V_S + (1 - \frac{n+2\alpha_H}{\alpha_Z(1-\alpha_H)})V_B$  (this is derived by simply plugging  $\alpha_B = n + \alpha_Z(1 - \alpha_H)$  into equation (3) and using this in  $p = qV_S + (1 - q)V_B$ ). This is clearly decreasing in  $n$  since  $V_S < V_B$ .

But from the text, we know that at these prices, the feasible pair with  $\alpha_B = \alpha_B^*$  (which corresponds to  $n = -(\alpha_Z(1 - \alpha_H) - \alpha_B^*)$ , and which we denote hereafter as  $n^*$ ) results in greater expected trading profits for each trading quantity within the pair than any trading quantity in any other feasible pair. Thus, if the pair  $n^*$  receives positive mixing probability in equilibrium, no other feasible pair can also receive positive probability. Thus, it remains to prove that the equilibrium must involve the feasible pair  $n^*$ .

We prove the result by contradiction. Suppose that there is an equilibrium in which the pair  $n^*$  is not used, but some other feasible pair, say corresponding to  $n = \hat{n}$ , receives positive mixing probability in equilibrium. The price when the market maker observes  $n = \hat{n}$  must then equal  $p_{\hat{n}}$  as derived above. Now consider whether there are prices that can be set when the market maker observes the off-equilibrium order flow  $n^*$  that will prevent any deviation by H to one of the quantities in the  $n^*$  pair. If the price  $p_{n^*}$  is set, H will deviate since the  $n^*$  node offers the greatest profits given this pricing. If a price  $p < p_{n^*}$  is set, the profit to the “buy” element in the  $n^*$  pair will be higher than if  $p_{n^*}$  were the price, so deviation to that element must be optimal. Similarly, if a price  $p > p_{n^*}$  is set, the profit to the “sell” element in the  $n^*$  pair will be higher, so deviation to that element must be profitable.

**Proof of Proposition 2:** Given our assumption that  $\alpha_X + \alpha_H < \frac{1}{2}$ , the objective function (5) will be negative for any feasible  $\alpha_H > \frac{\alpha_Z}{2 + \alpha_Z}$ . Taking the first derivative of (5) with respect to  $\alpha_H$  and evaluating it at  $\alpha_H = 0$ , yields

$$(9) \quad \frac{\Delta v \alpha_Z (1 - \alpha_X + \alpha_X^2)}{8(\alpha_X - 1)^2} > 0.$$

Evaluating the derivative at  $\alpha_H = \frac{\alpha_Z}{2+\alpha_Z}$  yields

$$(10) \quad \frac{\Delta v(2 + \alpha_Z)(\alpha_Z + \alpha_X(2 + \alpha_Z))}{4\alpha_X(2 + \alpha_Z) - 8} < 0,$$

where the inequality follows from  $\alpha_X, \alpha_Z < \frac{1}{2}$ .

It remains to show that there is a unique maximum. This is accomplished by taking the second derivative of the objective function with respect to  $\alpha_H$ , and showing that it is negative over the relevant range given our assumptions (the details are omitted for brevity). The result for  $\alpha_X^*$  follows directly from our assumptions on  $c(\alpha_X)$ .

**Proof of Proposition 4:** For the first comparative static result, it suffices to show that the appropriate cross partial derivative is positive. With  $\alpha_X = 0$ , the derivative of the objective function with respect to  $\alpha_H$  is

$$(11) \quad \frac{\Delta v((\alpha_H - 1)^4 \alpha_Z^2 - 4\alpha_H^2(3 - 4\alpha_H + \alpha_H^2))}{8\alpha_Z(\alpha_H - 1)^4}.$$

Taking the derivative with respect to  $\alpha_Z$  then yields the cross-partial

$$(12) \quad \frac{\Delta v((\alpha_H - 1)^3 \alpha_Z^2 - 4(3 - \alpha_H)\alpha_H^2)}{8\alpha_Z^2(\alpha_H - 1)^3} > 0.$$

For the second comparative static result, note that when setting (11) equal to zero with  $\alpha_X = 0$  and solving for  $\alpha_H$ ,  $\Delta v$  will drop out of the solution.

**Proof of Proposition 5:** With  $c(\alpha_X) = 0$ , votes are free. It is easy to see that the value wedge H can create,  $V_B - V_S$ , is maximized when H can completely determine the outcome. Furthermore, its maximized value must equal  $\Delta v$ . Thus, H will acquire at least  $\frac{1}{2}$  of the votes, and the value wedge will no longer depend on  $\alpha_H$ . H's maximization problem with respect to  $\alpha_H$  is therefore given by

$$(13) \quad \max_{\alpha_H} \frac{(\Delta v)(\alpha_Z^2(1 - \alpha_H)^2 - 4\alpha_H^2)}{8\alpha_Z(1 - \alpha_H)}.$$

It is easy to show that this is decreasing in  $\alpha_H$ .

**Proof of Proposition 6:** First assume that H's problem has an optimum at some  $\alpha_H^* < \frac{\alpha_Z}{2+\alpha_Z}$ . Evaluating the objective function (8) and taking the derivative with respect to  $\alpha_H$

yields

$$(14) \quad \frac{\Delta v [(\alpha_H - 1)^4 \alpha_Z^2 - 4\alpha_H(\alpha_H - 1)(4\alpha_h + \alpha_H(\alpha_H - 3))]}{8\alpha_Z(\alpha_H - 1)^4}.$$

Taking the second derivative with respect to  $\alpha_H$  yields

$$(15) \quad \frac{\Delta v(2\alpha_h + \alpha_H(4\alpha_h - 3))}{\alpha_Z(\alpha_H - 1)^4}.$$

This will be positive for all  $\alpha_H \in [0, \frac{2\alpha_h}{3-4\alpha_h}]$  but negative for higher  $\alpha_H$ . Thus, the objective function is either always convex or convex then concave as  $\alpha_H$  rises from zero over the relevant range.

Evaluating (14) at  $\alpha_H = 0$  yields  $\frac{\Delta v \alpha_Z}{8} > 0$ . Evaluating it at  $\alpha_H = \alpha_h$  yields

$$(16) \quad \frac{\Delta v(\alpha_Z^2(\alpha_h - 1)^3 - 4\alpha_h^2(1 + \alpha_h))}{8\alpha_Z(\alpha_h - 1)^3} > 0.$$

Thus, if there is an interior solution, we must have  $\alpha_H^* > \alpha_h$ .

Now consider the possibility that the optimal solution has  $\alpha_H \geq \frac{\alpha_Z}{2+\alpha_Z}$ . In this case, H will always find it optimal to vote in favor of increasing firm value (the probability of going short and voting to decrease value goes to zero as  $\alpha_H$  approaches  $\frac{\alpha_Z}{2+\alpha_Z}$ ). But since shares prior to the record date sell at their expected value, if H expects to always vote the right way, it must be optimal to acquire enough shares so that H's voting power is at least  $\frac{1}{2}$ . The expected payoff to this strategy, the "always vote yes" strategy, equals the expected increase in the value of H's initial stake,  $\alpha_h$ , due to the higher probability of a correct decision.

Now consider the overall equilibrium. From the previous analysis, we know that when  $\alpha_h = 0$  there is a strictly interior maximum with  $\alpha_H < \frac{\alpha_Z}{2+\alpha_Z}$ . Thus, for small enough  $\alpha_h$  this will still be true. As  $\alpha_h$  increases, the optimal  $\alpha_H$  must increase (the cross partial derivative of the objective function with respect to  $\alpha_H$  and  $\alpha_h$  equals  $-\frac{2\Delta v \alpha_H}{\alpha_Z(\alpha_H - 1)^3} > 0$ ). Thus, at some  $\alpha_h < \frac{\alpha_Z}{2+\alpha_Z}$ , say  $\alpha_h^*$ , we will have  $\alpha_H^* = \frac{\alpha_Z}{2+\alpha_Z}$ . Now note that at that point, H will always vote yes, but does not have enough voting power to swing the election, thus the "always vote yes" strategy where it instead acquires enough shares to reach  $\alpha_H$  must be better. Since the payoff to this strategy is zero at  $\alpha_h = 0$  and increasing in  $\alpha_h$ , there must be some  $\hat{\alpha}_h \in [0, \alpha_h^*]$

where the profit of the “always vote yes” strategy equals the profit of the interior trading strategy, which proves the result.

**Proof of Proposition 7:** The result follows directly from the fact that, for any  $\alpha_h < \alpha_h^*$  and  $\alpha_H$ , the profitability of playing the mixed strategy increases in  $\alpha_Z$ , while the profitability of the “always vote yes” strategy is not affected by  $\alpha_Z$ .

**Proof of Proposition 9:** As in Proposition 5, it is easy to show that when votes are free, H will always buy enough to have voting power of at least  $\frac{1}{2}$ . From the results above, it is immediate that if  $\alpha_h \geq \frac{\alpha_Z}{2+\alpha_Z}$ , H will find it optimal to simply vote to increase the value of the firm and enjoy the gain in the expected value of its shares. Thus, there is no incentive for trading, which proves part (c).

Now consider case with  $\alpha_h < \frac{\alpha_Z}{2+\alpha_Z}$ . Here, H can swing the election, so if it chooses to play the mixed strategy, it can maximize the value wedge at  $V_B - V_S = \Delta v$ . Its objective function, assuming a stake  $\alpha_H < \frac{\alpha_Z}{2+\alpha_Z}$ , is then

$$(17) \quad \max_{\alpha_H} \frac{\Delta v (\alpha_Z^2 (1 - \alpha_H)^2 - 4\alpha_H^2)}{8\alpha_Z (1 - \alpha_H)} + E[\alpha_h (q^* V_S^* + (1 - q^*) V_B^*)].$$

It is easy to show that this objective is concave over the relevant range. The derivative with respect to  $\alpha_H$  at  $\alpha_H = 0$  is  $\frac{\alpha_h \Delta v}{\alpha_Z} - \frac{\Delta v \alpha_Z}{8}$ , which is negative for all  $\alpha_h < \frac{\alpha_Z^2}{8}$  and positive for all greater  $\alpha_h$ . Finally, it is easy to show that the derivative is negative at  $\alpha_H = \alpha_h$  for all  $\alpha_h < \frac{\alpha_Z}{2+\alpha_Z}$ , which proves parts (a) and (b) *assuming* an interior solution. The final step is to prove that the interior solution exceeds the payoff to always voting the right way – but this must be true since, in this case, the vote outcome will be determined by H either way, so there is no discrete difference in payoffs for setting  $\alpha_H = \frac{\alpha_Z}{2+\alpha_Z}$  and always voting yes versus setting  $\alpha_H = \frac{1}{2}$  and always voting yes.

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